Government formation as logrolling in high-dimensional issue spaces

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ABSTRACT

Analytical models of government formation typically assume low-dimensional real policy spaces. Behaviorally, however, politicians negotiate to form governments in high-dimensional discrete issue spaces. We model these negotiations, leveraging the fact that different politicians typically attach different importance to the same issue, allowing gains from trade when they negotiate agreed positions on large packages of issues. The set of issues in an agreed package is endogenous; politicians need not agree on every issue before they go into government together, "tabling" issues on which they agree to disagree. We exercise our model computationally, calibrating it to 91 real-world government formation settings, and mapping out the relative probability of Condorcet winning cabinets in different settings. This probability measures how hard it is for negotiators to find Condorcet winning cabinets in a giant high-dimensional state space. We test this claim empirically with a statistical model of the duration of negotiations after an election.

Keywords: government formation, logrolling, discrete issue spaces, bargaining delays

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MOTIVATION

The German federal election of 24 September 2017 inflicted major losses on both parties in the governing "grand coalition" of Christian Democrats (CDU/CSU) and Social Democrats (SPD). While these parties still controlled a legislative majority, they were widely seen as having "lost" the election. The SPD immediately announced it would not participate in a new coalition cabinet and would return to opposition. Angela Merkel, incumbent Chancellor and leader of the CDU, began months of negotiations with Free Democrats (FDP) and Greens. These negotiations confronted huge policy differences between very different parties and were ultimately unsuccessful. The SPD then reversed its stance and entered negotiations with CDU/CSU in early January 2018. Senior politicians from both parties began lengthy negotiations. These resulted in an agreement to form a coalition cabinet, which was approved by the membership of the SPD on 4 March 2018. What were these politicians, who were already partners in government, *doing* in there, behind closed doors, during those weeks of intense negotiations?

The most obvious end product of their endeavors was a 177-page joint policy program setting out policy agreements between coalition partners under 59 different headings. Getting to this point clearly involved fine-grained discussions of the many points of difference between the detailed and divergent policy programs each party had just pitched to voters at election time. These programs were themselves extensive: the 2017 CDU election manifesto ran to 76 pages, the SPD manifesto to 116 pages. These government formation negotiations somehow mashed together two long, detailed and divergent party policy programs into one very long joint program of government. Clearly, this could not be achieved overnight.

In what follows, we develop a behavioral theory of how this might have happened, characterizing these negotiations as "logrolling" in high dimensional discrete issue spaces. We exercise our model computationally, using a suite of empirically calibrated simulations, before returning to the empirical question motivating this paper: why do some government formation negotiations last much longer than others? Before doing any of this, however, we sketch the existing intellectual landscape in which most current models of government formation are located, and then sketch the somewhat different landscape in which we locate our own theoretical approach.

THE CURRENT LANDSCAPE:

LOW-DIMENSIONAL *ANALYTICAL* MODELS OF GOVERNMENT FORMATION

There is a substantial analytical literature on government formation in parliamentary democracies where no party commands a legislative majority. The first generation of this intellectual project, associated with early work by Riker (1962), evolved within the traditions of social choice theory and focused on the allocation of a fixed set of "perks" of office, such as cabinet portfolios (Gamson 1961; Banzhaf 1965; Leiserson 1966; Browne and Franklin 1973; Dodd 1976; Einy 1985; Schofield and Laver 1985). Subsequent work shifted focus to the need to reconcile conflicting policy positions, describing these positions using simple one-dimensional or at most very low-dimensional *latent* "policy" spaces. The latent dimensions summarize politicians' correlated positions on the much large number of *manifest* issue dimensions describing the actual concerns of politicians on the ground (Stokes 1963; Converse 1964). Agreed government policy programs were treated as the social aggregation, in these low-

dimensional latent spaces, of the conflicting preferences of government members. The aim was to identify equilibrium governments in an environment essentially devoid of institutions, save for the binding constitutional constraint defining all parliamentary democracies: the requirement that the incumbent government maintains majority support in the legislature (Axelrod 1970; deSwaan 1973; Grofman 1982; Luebbert 1986; Strom 1990). While not explicitly concerned with government formation, the McKelvey-Schofield (1986, 1987) "chaos theorems" confirmed the generic instability of majority decision-making in multidimensional policy spaces, given this institution-free world.

Solutions to the problem of generic instability emerged within the theoretical context of a "new institutionalism" which associated political equilibria with institutional features of the empirical context under investigation. One approach leveraged the institution of ministerial government to specify "portfolio allocation" models of policy-based government formation (Austen Smith and Banks 1990; Laver and Shepsle 1990, 1996). Equilibrium governments emerge in such models because politicians are assumed to believe that government policy on any latent policy dimension will be the ideal policy of the minister with formal jurisdiction over this dimension. This superimposes a finite lattice on the real policy space and the resulting discrete set of possible cabinets renders government equilibria possible, even in multidimensional policy spaces. Another approach harnessed a canonical non-cooperative "alternating offers" bargaining model (Rubenstein 1982) to "bargaining in legislatures" (Baron and Ferejohn 1989) both over fixed perquisites (Ansolabehere et al 2005; Laver et al, 2011) and

over policy (Baron 1991; Duggan 2000; Baron and Diermeier 2001).[1] This assumes that government formation negotiations are institutionally constrained by an *exogenously imposed bargaining protocol*, which specifies, *inter alia*: which politicians can make offers to which other politicians; the order in which offers can be made; that offers are made in public not in private; that no side-payments are allowed between politicians. At the expense of empirical realism, the imposition of an exogenous protocol such as this[2] allows the specification of a non-cooperative government formation game, which can be solved for equilibrium by backwards induction.[3]

The current theoretical canon concerning government formation is essentially *analytical*, abstracting from the nitty-gritty behaviors which determine political outcomes on the ground. A critical feature of this abstraction is the near-universal description of politicians' preferences in terms of a few *latent* policy dimensions. Policy negotiations involved in forming an *n*-party coalition are analyzed "as if" these negotiations discuss how to resolve *n* different common knowledge ideal points in some low-dimensional latent space into a single coalition policy point in that same space. Crudely speaking in one-dimensional terms, negotiations are characterized as if Jack announces he's a 5 on the left-right dimension, Jill announces she's a 2,

[1] See Humphreys (2008) for an excellent analysis of the relative merits of social choice and institutionalist approaches to analyzing coalition bargaining.

[2] Morelli (1999) proposes an alternative protocol in a "demand bargaining" model that often generates different equilibria from those generated by the Baron-Ferejohn protocol.

[3] Actors in these models are sharply constrained strategically; i.e., they cannot engage with multiple offers simultaneously, they cannot logroll, etc.

and they settle on 3.5 as their joint policy position. This abstract characterization offers no *behavioral* account of what actually happens during these negotiations and no explanation of the striking empirical regularity that it takes much longer to find a common position in some settings than in others.[4]

A HIGH-DIMENSIONAL LANDSCAPE FOR GOVERNMENT FORMATION

Gains from trade

None of the standard models of government formation addresses the plain fact that negotiating politicians must reconcile two or more lengthy, specific, and conflicting party policy programs into one joint program of government. In behavioral terms, these negotiations involve discussions between senior politicians about the specifics of *manifest* issues, not the generalities of *latent* policy dimensions.

> *Assumption 1:* The behavioral environment for government formation is a *high-dimensional space of manifest issues*.
>
> *Assumption 2:* Negotiators typically have different ideal positions on, and attach different levels of importance to, these issues.

Since different negotiators may attach different levels of importance to the same issue, they can unlock gains from trade during government formation negotiations. They may make

[4] Golder, et al. (2012), treat proposals as essentially random in an effort to set a baseline expectation. But, see also Martin and Vanberg (2014).

concessions on less-valued issues in exchange for reciprocal concessions on more-valued issues. We therefore characterize policy negotiations during government formation as *logrolling*. Scholars have devoted attention to logrolling over *legislation*, typically but not exclusively in the US Congress. This may involve either a diverse set of different bills[5], or a diverse set of issues treated by a single bill (Stratmann 1992, Carrubba and Volden 2000, Stratmann 2004, Hortala-Vallve 2011, Aksoy 2012). In a rare application of the notion of logrolling to government formation in parliamentary democracies Falco-Gimeno, referring to a small number of latent policy dimensions, argues that logrolling is "a simple way to resolve partners' differences in emphasis" (Falco-Gimeno 2014: 344). Agreement is possible when negotiators' policy preferences are "tangential" (Leubbert 1976; Ecker and Meyer, 2017), meaning precisely that different negotiators attach different levels of importance to the same issues. Logrolled policy agreements between negotiators with tangential preferences are self-enforcing, since coalition partners can cede control of less-valued policy dimensions to ministers from other parties. This contrasts with deals between partners whose preferences are not tangential, which require control mechanisms to monitor and constrain the potential for ministerial drift.

The authors referred to above discuss gains from trade arising from logrolling but *do not model logrolling behavior*, whether in drafting legislation or in forming governments. Particularly in relation to government formation, low-dimensional representations of inter-party bargaining do not allow us to model the fine-grained horse-trading behavior that

[5] In this case it may be referred to as "vote trading".

reconciles detailed and conflicting party policy programs into an agreed program of government. This behavior takes place in a high dimensional manifest issue space.

Tabled issues

It is almost never essential that an agreed joint program deals with every single issue in each partners' manifestos. Negotiators may resolve disagreement on a hundred issues, for example, leaving a small number of issues about which they all feel strongly, yet disagree. This does not mean that they can never go into government together. They can instead "agree to disagree" on matters of residual conflict, in which case we say they "table" these issues.

> *Assumption 3.* Negotiating politicians may agree to *table* issues on which they agree to disagree, leaving these issues out of the joint program altogether.

The set of agreed issue positions in the joint program of any cabinet is therefore *endogenous to policy negotiations* between partners. When coalition partners table issues they in effect hope that unforeseen events will not force them to negotiate an agreed position on an issue on which they manifestly disagree. A terrorist attack may generate the need for divisive choices on civil liberties, for example, or a long drought may force hard choices on environmental policy.

The possibility of what we call tabled issues is discussed in recent work by Bowler et al (2016) on the role of formal dispute resolution mechanisms in written coalition agreements. Given time pressure during negotiations and prospective coalition partners' diverse policy preferences, partners may decide to leave some issues to be decided later, rather than agreeing on everything at the time the government is formed.

To get an intuitive feel for the set of tabled issues, consider a two-party prospective coalition and a set of issues for which there is a perfect rank order correlation between the issue saliences of coalition partners. Since both partners agree perfectly on which are the high and low salience issues, they have no incentive to trade positions on issues in this set. All such issues must be tabled. In contrast, if coalition partners' saliences for some set of issues are *inversely* correlated, then it will be easy to find logrolling gains from trade. The residual set of tabled issues in a joint program of government therefore contains issues rated as important by most or all coalition partners. The more such issues are tabled, the less likely a prospective coalition is to form compared to prospective coalitions with fewer tabled issues. If such a coalition does form, the associated cabinet will be more vulnerable to shocks that force a divisive cabinet decision on some salient issue and, for this reason, will have a more uncertain future.[6] While the focus of this paper is on the effect of logrolling on the duration of bargaining over government formation, the size of the set of tabled issues may also have a bearing on cabinet stability, a possibility to which we will return in future work.

LOGROLLING BEHAVIOR

Our general claim is that, given divergent ideal issue positions in a high dimensional issue space but faced with the need to agree a joint program of government, negotiating politicians seek

[6] Of somewhat less interest, the set of tabled issues also contains "boring" issues, rated as unimportant by most or all coalition partners. Boring tabled issues seem unlikely to pose a threat to cabinet stability.

Pareto optimal trades of issue positions. Jack agrees to Jill's proposal for tough new regulation of financial services (with which Jack disagrees but cares about less strongly than he cares about automobile emissions) in exchange for Jill accepting Jack's tough new regulation on auto emissions (with which Jill disagrees but cares about less strongly than financial services). This "logroll" generates a Pareto improvement for them both. Assumptions 1 – 3 above are general and consistent with many possible types of logrolling behavior. An intriguing case study of a negotiation in precisely these terms over the joint issue program of a real coalition cabinet, the 2010 Conservative-Liberal Democrat coalition in Britain, can be found in Quinn et al (2011).

In order to gain enough traction to develop a general model of logrolling behavior, we now make three behavioral assumptions. The first is that politicians' issue positions are binary. The second is that politicians engaged in logrolling use simple behavioral rules rather than hyper-rational strategic analysis. The third is that the logrolls they realize are bilateral. Each of these assumptions may obviously be relaxed or modified in future work, but for now they allow us to build a tractable baseline model.

Binary issue positions

> *Assumption 4:* Politicians' ideal positions on key issues can be described as binary, reflecting whether they support (1) or oppose (0) a specific change to the status quo.

We characterize politicians' issue positions as discrete rather than continuous, not only to simplify our argument, but because actual joint programs for government are not vague low-dimensional statements of "policy", but sets of discrete proposals on particular issues. While in

theory many issues admit a "continuous" range of potential resolutions, in practice every issue is ultimately resolved in an up-or-down decision on some discrete proposal. The discrete issue space we envisage describes where politicians stand on this "final passage" decision. Do they promise to vote aye or nay on a particular proposal? Thus while tax policy, for example, can be *described* in continuous terms, actual *behavior* that changes the tax regime ultimately involves an up or down vote on some specific tax proposal.

While our assumption of binary issues positions is unusual for models of party competition and government formation, it is not unique. Krasa and Polborn (2010), for example, specify and analyze a "binary policy model", which they argue (p. 663) is "a quite realistic description of political campaigns". In their model, some issue positions for each politician are fixed exogenously and some are endogenously fungible as part of the process of electoral competition. We differ here by modeling government formation, not elections, and by assuming *all* issue positions are fungible. What we share with Krasa and Polborn is that it is the discrete nature of the binary issue space (not logrolling *per se*) which opens up the possibility of equilibrium as opposed to chaotic cycling – in our case the possibility of Condorcet winning joint programs of government. While it is easily possible to envision alternative logrolling models that assume continuous issue spaces, these are not our concern in this paper.

Non-strategic behavior

> *Assumption 5:* Politicians engaged in logrolling deploy behavioral rules of thumb rather than fully strategic analysis.

Logrolling in a high-dimensional discrete issue space is a complicated business. Our motivating example of government formation in Germany suggests that there might be as many as 100 discrete issue positions to resolve. This generates a state space of 2^{100} possible joint programs for government – a very large number.[7] If there are six legislative parties, as in the German case, there are 63 possible non-null coalitions, for each of which there is a probability distribution of expected joint programs drawn from this large state space. This suggests to us that politicians address the logrolling problem using behavioral rules, examples of which we specify below, rather than complex and likely intractable strategic analysis.[8] This does not of course preclude the possibility of subsequently developing a fully strategic model of logrolling in high-dimensional discrete issues spaces.

Simple bilateral logrolls

Assumption 6: Politicians consider only simple bilateral logrolls

We focus here on the simplest type of logroll, in which two parties trade positions on two issues. To get a sense of this imagine three party leaders A, B and C, with binary positions on three issues as follows: A(**1**, 0, 0); B(0, **1**, 0); C(0, 0, **0**). Issues shown in bold underline are highly

[7] 1,267,650,600,228,229,401,496,703,205,376 to be precise.

[8] On our assumptions politicians engaged in logrolling do not for example take account, when they agree to some trade in some coalition, of trades that might be done with other partners in other potential coalitions.

valued, other issues are valued much less. Without the possibility of logrolling, party leader A is likely to see C as the "closer" coalition partner, disagreeing on only a one issue rather than two. However, while A would simply like C to switch position on the first issue, it has nothing to offer in exchange. A and B, in contrast, can *trade* positions on the first two issues, with A conceding on the second issue in exchange for B conceding on the first. This is a simple bilateral logroll. The logrolled joint program (1, 1, 0) is a Pareto improvement for both partners and better for A than anything that could be achieved in coalition with C.

Simple bilateral logrolls are the low-hanging fruit of the issue trading environment. It is of course possible to imagine more complex logrolls, especially in large coalitions. Rather than two parties swapping positions on pairs of issues, more parties may agree to a larger package of trades across a set of issues and thereby realize a Pareto improvement. We leave this possibility for future work, noting that there will be many levels of complexity to consider once we move beyond simple bilateral trades, ranging from one-for-two to many-for-many, so this will not be a straightforward problem.

We also do not consider situations where parties trade policy for perks. In the above example, one could argue that C might well join a coalition with A by agreeing to (1,0,0) in exchange for some non-policy concession like a juicy government job. While this may in fact happen empirically, we leave trades of this kind for future work given they require much more complex behavioral rules.

We thus focus here on a subset of the possible logrolls between negotiating politicians. As we show below, however, this still gives us substantial purchase on explaining the duration of bargaining after an election. In large part this is because, as we also show when reporting

our results, the subset of bilateral logrolls is typically a high proportion of the universe of possible trades when politicians disagree on both issue positions and issue saliences – noting that when they agree on either of these matters, there is no scope for logrolling.

Formal behavioral model

Having set out our general assumptions, we now specify a behavioral model of logrolling consistent with these. We do not propose this model as anything like the last word on the subject, but do show that we can make good progress analyzing government formation using a simple behavioral model of logrolling in high-dimensional issue spaces.

Environment

The negotiators for each of n disciplined legislative parties (1 … i … n) are party leaders or their proxies, with relative weights specified by their parties' legislative seat shares. The legislature has a simple majority decision rule for choosing governments. There is an m-dimensional discrete space of binary preferences on each of a large set of issues expected to require action by any incoming cabinet. Negotiator i's ideal position is given by an issue vector P_i of size m, each element, p_{ij}, of which is a binary number reporting i's preference on issue j, which for now we take to be common knowledge.[9] Positions on each issue describe whether the relevant

[9] We will return in future work to consider logrolling in settings in which politicians' issue preference are private information, noting that nearly all conventional models of government formation assume politicians' policy preferences are common knowledge.

party leader supports (1) or opposes (0) a proposed change to the status quo. The relative salience of each issue for politician *i* is given by a vector S_i of size *m*. Element s_{ij} shows the relative salience of issue *j* to politician *i:* $0 \leq s_{ij} \leq 1$ and $\Sigma_j (s_{ij}) = 1$. Negotiations between prospective coalition partners are aimed at generating an agreed joint issue vector for the proposed government coalition *c*. Elements of this, p_{cj}, state the agreed coalition position on issue *j*. The utility U_{ic} for politician *i* from cabinet *c* decreases linearly in the salience-weighted Hamming distance between cabinet policy and politician *i*'s ideal point: $U_{ic} = -\Sigma_j s_{ij} \bullet |p_{ij} - p_{cj}|$.[10]

The possibility of tabling an issue in the joint program generates uncertainty about what will happen if an exogenous shock forces cabinet members to confront this issue. Politicians may be more, or less, tolerant of this uncertainty, so we specify an agent-specific parameter τ_i ($0 \leq \tau_i \leq 1$), which reflects the attitude of politician *i* to the uncertainty associated with tabling an issue. If $\tau_i = 0$, we specify the most "optimistic" (or risk tolerant) politician possible, who expects no policy loss to arise from tabling an issue. If $\tau_i = 1$, on the other hand, we specify the most "pessimistic" (or risk averse) politician, who *always* expects a tabled issue to turn out as badly as possible, yielding the maximum policy loss. Thus the salience-weighted expected value of any tabled issue, $|p_{ij} - p_{cj}| = \tau_i$,, appears in each agent's utility function in precisely the same way as for any other issue.

[10] We will return in future work to setting in which agents derive utility both from government policy outputs and from the perks of office.

Logrolling behavior

We describe government formation as a two-stage process. The first stage resolves "uncontested" issues, for which all negotiating parties have the same preferred position. We assume that politicians negotiating proposed cabinet *c* simply copy their unanimously agreed positions on uncontested issues into their joint issue vector P_c.[11] Trivially, this means that the issue vector of any single-party cabinet is the ideal point of the party concerned.

The second stage involves bilateral logrolling. The behavioral rule we specify for this takes as given, for a particular "negotiation attempt", a particular ordering of contested issues for discussion. Since we do not propose an exogenous agenda setter for government formation negotiations, our stochastic model randomizes the issue agenda over many trials. Different negotiation attempts may result in different logrolls, with different probabilities that the outcome will be preferred by a majority of legislators. Our stochastic model therefore maps out, for any given setting, a *distribution* of predicted logrolling outcomes over many trials.

For each of the *r* iterations of our model for a given setting, with each iteration associated with a different random draw of agents' issue positions, issue saliences, and the issue agenda, our logrolling algorithm proceeds at follows for each possible coalition in the setting under investigation:

i. Identify all contested issues for which one proto-coalition member is a *unique outlier*, with a position opposite to that of all other members. A unilateral change of position by the unique outlier on such an issue results in unanimous agreement between coalition

[11] Formally, if $p_{ij} = 0 \; \forall \; i \in c$, then $p_{cj} = 0$; if $p_{ij} = 1 \; \forall \; i \in c$, then $p_{cj} = 1$; else p_{cj} = "contested".

partners. All contested issues in two- and three-party coalitions must have a unique outlier; in coalitions with four or more parties, issues may have no unique outlier.[12]

ii. Mark contested issues with no unique outlier as "tabled", since no simple trade can resolve these. Mark other contested issues as "tradable".

iii. Treating the ordering of issues as exogenous, and recalling that we randomize this order over many trials, consider negotiator A, the unique outlier on the first tradable issue, *x*.[13] Negotiator A looks down the list of tradable issues for which *other* members are unique outliers, looking for the first such issue, *y*, with a unique outlier, B, from one of the n-1 other parties, such that A *values y more than x*, but B *values x more than y*.[14] Negotiator A in effect looks for a counterparty for A's low-value position on issue *x* willing to flip on A's high value issue *y* in exchange for A's flip on issue *x.* If there is such an issue, and there may not be, then A and B trade positions on issues *x* and *y*. Following this trade, $p_{cx} = p_{ay}$ and $p_{cy} = p_{bx}$.

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[12] In the most-recent release (3.0) of the *European Representative Democracy Dataset* (www.erdda.se), 69 percent of the 429 postwar European coalition cabinets listed comprised two or three parties, while the 31 percent comprised four or more parties.

[13] In two-party coalitions, both agents are unique outliers on all contested issues. In this case we schedule negotiators by party size when reviewing tradable issues for potential logrolls.

[14] This is therefore a "greedy" logrolling algorithm. Agents realize the first profitable trade they encounter, rather than analyzing the universe of all possible trades in search the most profitable. Clearly, as we have noted, other algorithms are possible.

iv. Iterate the logrolling process, moving down the list of tradable issues until no such trade is possible.

v. The residual set of contested issues for which no simple logroll is available is tabled by the coalition partners with costs $\boldsymbol{\tau}$.

After *r* iterations of the model there is, for each possible coalition in each government formation setting under investigation, a distribution of *r* logrolled joint issue vectors, drawn from the finite state space of possible issue vectors. These distributions of joint issue vectors are the output of our behavioral model of logrolling between party negotiators.

GOVERNMENT FORMATION

We now face a problem having nothing whatsoever to do with logrolling. Given a distribution of policy expectations associated with each possible cabinet in a given setting, how do party leaders go about forming a government?

Considering the huge stakes involved in the formation of national governments, we see no reason to believe that senior politicians feel in any way constrained by enforceable "rules of the game" which set out, for example: when they can speak; to whom they can speak; what sorts of thing they can say; and whether this happens in public or in private. In our view, government formation involves freewheeling negotiations between senior politicians, not a well-ordered conversation conforming to exogenously enforced rules. This leads us to:

Assumption 7: Government formation takes place in an environment in which negotiators can say whatever they choose, to whomever they choose, whenever they choose to say it, in public or in private.

We think of this as an "open outcry" system, as opposed a tightly-structured game that can be solved by backwards induction. Rather than making a point prediction, therefore, we map out the *distribution of possible outcomes*, according to our model, for a given setting and model parameterization. We do this for each setting under investigation in a set of *r* trials, or "government formation attempts". For each trial in each setting, we use the logrolling algorithm set out above to derive a joint issue vector for each arithmetically possible cabinet. For each trial we then compute the outcome of the majority rule tournament involving every possible pairwise comparison between the $2^n - 1$ possible non-null cabinets. We find that this tournament may sometimes yield a Condorcet winning cabinet, one that is majority-preferred to any alternative.[15] Aggregating the results from the *r* trials in a given setting, we then map out the probability, conditional on model parameters, that each of the arithmetically possible

[15] In a result not reported here because it did not substantially alter our conclusions, but available from the authors, we also computed the *uncovered set* generated by the tournament over the set of possible coalitions. As Penn notes, "[t]he uncovered set has been of particular interest to social scientists because it has been observed that generally, and under many different institutional arrangements, strategic behavior by voters leads to outcomes in the uncovered set" (Penn 2006 p. 83). We did however encounter a commonly noted drawback of the uncovered set: it is often large and therefore uninformative.

coalitions in a given setting is Condorcet winning. Summing these probabilities over the set of possible coalitions, this gives an estimate of aggregate probability of a Condorcet winning cabinet in this setting, given the expected joint issue vectors generated by our logrolling model.

In mapping out the probability of Condorcet winning cabinets in this way, conditional on model parameters, we are performing the computational equivalent of analytical comparative statics.[16] It is important to recognize that we are not offering a behavioral model of how politicians might *search* for candidate cabinets within the large and complicated universe of coalition possibilities. Neither do we offer a model of how such cabinets, once identified, are *installed* into office. These are important matters to which we will return in future work.[17] Our mapping of the probabilities of Condorcet winners is in effect an analytical device. It highlights the relative vulnerability, *ceteris paribus*, of a given candidate cabinet to any alternative that might be proposed. This is informative since, whatever search behavior politicians might

[16] Results derived from our model differ from those derived from non-cooperative game theoretic models of coalition formation. We do not provide proofs of equilibrium outcomes, but instead deploy applied statistics on results from a Monte Carlo of our model to analyze the role of theoretically central parameters across a range of assumptions. For example, in a finding directly analogous to deductive comparative statics, we show that the probability of finding a Condorcet winner increases monotonically in the dimensionality of the issue space, and that this holds across the entire range of other choices in our model.

[17] We are not aware of any other author who has modeled this search process.

actually use, and whatever government formation process is envisaged, Condorcet winning cabinets will on almost any account be the hardest to beat.

Importantly, we use our measure of the probability of Condorcet winning cabinets to investigate why some government formation settings are associated with substantially longer bargaining delays than others. Our main empirical hypothesis, set out below, is that the duration of bargaining after an election is a function of the complexity of the bargaining environment. In settings where more Condorcet winners can be formed a result of successful logrolls, it will take fewer government formation attempts to search the huge state space for a Condorcet winning cabinet and once found, these cabinets should be proof against alternative proposals. This in turn implies that, all else equal, bargaining delays should be shorter. In environments with few Condorcet winners in the huge multidimensional state space we analyze, finding a Condorcet winner can be like finding a needle in a haystack.

MODEL CALIBRATION

Calibration strategy

As noted, we use Monte Carlo simulations to map out distributions of Condorcet winning cabinets according to our model, conditional on model parameters, for any setting under investigation. We know *a priori* that there will be more uncontested issues the more negotiators' issue *positions* are correlated. There will be fewer potential trades the more negotiators' issue *saliences* are correlated across the set of contested issues. The structure of negotiators' issue preferences therefore has a large impact on logrolling. If we were to assign negotiators' issue positions and saliences randomly, then this would be very unrealistic and

would make it difficult to test our model empirically. But what is a "realistic" structure for politicians' issue preferences in the settings we investigate?

This question concerns the empirical calibration of our model to real political settings. Rather than making abstract decisions about issue preferences, we calibrate these to specific real-world settings for government formation. If Nature allowed us to observe politicians' preferences in the "true" high-dimensional issue space for each setting, then we could feed our model raw data. In practice we cannot do this. For each empirical setting, therefore, we *simulate* a distribution of *r* high dimensional discrete issue spaces, all of which are consistent with some "ground truth" information about the empirical setting under investigation. We start with widely available estimates of parties' low-dimensional policy positions and saliences in particular settings, treating these as authoritative summaries of the underlying structure of a distribution of possible high dimensional discrete issue spaces that might be realized in the same settings. We simulate a different *m*-dimensional issue space for each of the *r* trials by "shocking" the vector of party positions by adding zero-centered Gaussian noise to the underlying policy dimensions *m* times and rescaling the resulting shocked positions to be either 0 or 1. We also shock relative issue saliences for each party in the same way, this time rescaling the shocked saliences to sum to unity for each party across all issues. This gives us negotiators' positions and relative issue saliences on a set of *m* binary issue dimensions for each trial, each with an underlying structure consistent with our low dimensional empirical summaries. We set out our method in Appendix B and our computer code is available at *http://sdemarchi.weebly.com/*.

Calibration data

The empirical settings we investigate are Western European parliamentary democracies in which it is typical for no single party to control a legislative majority. Our calibration data come from four expert surveys: Laver and Hunt (1992); Benoit and Laver (2006), and the Chapel Hill Expert Surveys (CHES)[18] of 2006 and 2010. While individual surveys collected data on an eclectic set of policy dimensions, all of these surveys estimated party positions and relative saliences for five latent policy dimensions: economic policy (taxes vs spending); social policy (liberal vs conservative); environmental policy; decentralization policy; immigration policy. We used these five latent policy dimensions to seed the stochastic generation of high dimensional issue spaces, in the manner described above.

Given the exploding size of the universe of possible coalitions, and even more rapidly exploding size of the majority rule tournament as the number of political parties increases, we confine our analysis to settings with up to ten legislative parties. This excluded two countries (Belgium and Italy) where there were always more than ten legislative parties. We therefore generated Monte Carlo results for empirical model calibrations derived from the 11 countries listed in Appendix A, observed at four expert survey time points.[19] There were occasionally two or three government formations in these countries in a single period between elections, resulting in a total of 91 different calibrated government formation settings.

[18] https://www.chesdata.eu/

[19] Cyprus was also excluded given its very unusual constitutional history.

EXERCISING THE LOGROLLING MODEL

The distribution of *r* high-dimensional discrete issue spaces we generate for each empirical setting fulfils two distinct roles. From a practical point of view, absent data on parties' issue positions and relative saliences in high-dimensional issue spaces, we generate distributions of *possible* high dimensional spaces consistent with low-dimensional descriptions of these settings for which we do have data. Much more important from a theoretical perspective, we model the process of government formation as *stochastic* rather than deterministic, and the random component in the generation of high dimensional issue spaces captures this. Technically, this has the effect of randomizing the issue agenda. From a deeper theoretical perspective, this models the fundamental uncertainty in the political environment by analyzing distributions of issue spaces that might arise in any given setting. Substantively, when we come to discuss the duration of bargaining in this stochastic environment, we interpret each of the *r* model runs as a "negotiation attempt", and use the relative number of negotiation attempts required to identify a Condorcet winner in a given setting taken as a measure of likely bargaining delays.

We designed the parametrization of our stochastic model as follows. We fixed the number of trials, *r*, at 1000 for each setting under investigation. Values of τ, the policy loss to negotiators from tabled issues, were drawn each trial from a uniform distribution on the (0, 1) interval.[20] Values of *m*, the dimensionality of the simulated issue space, were drawn each trial

[20] For this initial work, and lacking any empirical basis to do otherwise, we constrained all agents to have the same values of τ_i.

from a uniform distribution of integers on the (5, 500) interval. Two additional parameters arise as part of our stochastic computational model. As explained in Appendix B, these are σ_p, a policy noise parameter, and σ_s, a salience noise parameter. As σ_p and σ_s increase, the parties' issue positions and saliences become more random, less correlated, and less connected to the low-dimensional empirical summary of these. Each of these parameters was drawn for each trial from a uniform distribution on the (0.25, 0.50) interval. [21]

To sum up how we exercise our model for each of the 1000 simulated high-dimensional realizations of each of the 91 empirical settings under investigation, we first randomly sample model parameters as described above. We then generate the resulting *m*-dimensional binary issue space. Given this, we compute the policy logroll for each of the 2^n-1 non-null coalitions. We then implement a majority rule tournament of all possible pairwise comparisons between coalitions, identifying the Condorcet winner if one exists. This allows us to estimate to probability of a Condorcet winner in each setting, conditional on model parameters.

MAPPING THE PROBABILITY OF CONDORCET WINNING BILATERAL LOGROLLS

We first exercise the calibrated model by using it to map out the probability of Condorcet winning potential cabinets in real government formation settings, conditional on model

[21] We chose these intervals on the basis of exploratory simulations which showed, given low-dimensional data on party policy positions and saliences, that lower values generated "too little" variation in the high dimensional issue spaces we generated, leaving them all more or less the same, while higher values generated "too much" variation, leaving them all more or less random and unrelated to the underlying empirical setting.

parameters. This, as noted, is the computational analogue to analytical comparative statics. Recall that unconstrained weighted majority voting in multidimensional policy spaces is generically associated with chaotic cycling and no Condorcet winner. Recall too that, as noted above and found by previous authors such as Laver and Shepsle (1996), Krasa and Polborn (2010), the discrete nature of the issue space generated by our assumption of binary issue positions opens up the *theoretical* possibility of Condorcet winners. We are therefore interested in estimating the *empirical* probability, according to our model, of potential Condorcet winning cabinets in actual government formation settings.

Each of the government settings we investigate involves a particular *type* of legislature, according to a classification proposed by Laver and Benoit (2015).[22] The four types of minority legislature in which no single party has a majority are: (B) Single dominant party[23]; (C) Top three system[24]; (D) Top two system[25]; (E) "Open" legislature[26]. Different legislative types impose different theoretically important arithmetical constraints on the set of winning coalitions and

[22] We include these variables along with measures of the number of parties in a system as controls. Our results are robust to excluding these controls.

[23] First and third largest party control a majority between them; second and third largest do not.

[24] Second and third largest party control a majority between them.

[25] First and second largest party control a majority between them; first and third largest do not.

[26] First and second largest party do not control a majority between them; i.e. no winning two-party coalition

therefore have an impact on government formation. While this does not affect our model in any way, we analyze our results by legislative type. Table 1 shows the relative frequency with which our model predicts Condorcet winning cabinets:

Table 1: Probability of predicted Condorcet winner by legislative type

	Total
B: Dominant	.29
C: Top three	.34
D: Top two	.11
E: Open	.08
Total	.23

The bottom line from Table 1 is that our model of government formation as logrolling in high dimensional discrete issue spaces frequently but by no means invariably generates Condorcet winning potential cabinets. Across all settings, roughly one quarter of all logrolled potential cabinets were Condorcet winners, rising to one third of cabinets in "Top Three" legislatures, in which any two of the three largest parties may form a winning coalition, and Type B legislatures with a single dominant party. Condorcet winning potential cabinets were least likely to be predicted in "Open" legislatures, in which bargaining is more complex because every winning coalition must comprise at least three parties.

We also found that the relative frequency with which our model predicts Condorcet winning potential cabinets is sharply affected by model parameterization. Since these relationships are for some parameters non-linear, we report them graphically in Figure 1.

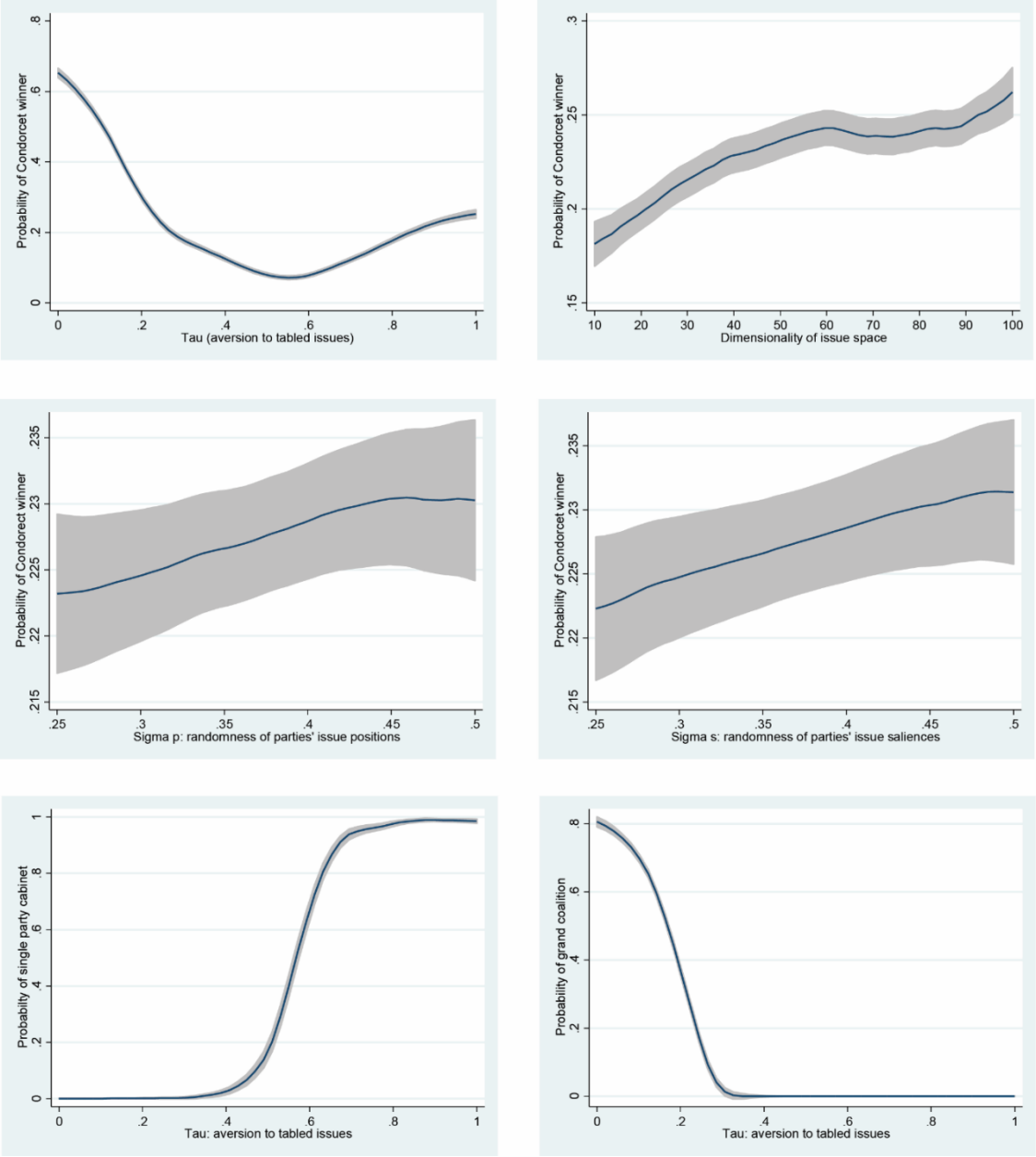


Figure 1: Fractional polynomial fit plots of the relationship between model parameters and predictions of Condorcet winning cabinets (95% confidence intervals shaded gray)

The top left panel shows that Condorcet winners predicted with substantially higher probability at extreme values of τ, dipping to a lower level in the mid-range, modeling risk-neutral negotiators. When τ is low, we model optimistic or risk-seeking negotiators who expect to prevail should a shock force a tabled issue onto the political agenda. Tabling an issue does

not create an anticipated policy loss for such politicians. This has the striking effect that low τ politicians tend to favor coalitions with more parties, including grand coalitions of all parties, since these tend to have more tabled issues. While this is not self-evident, one finding from exercising our model is that the probability of potential Condorcet winning cabinets increases when politicians are risk-seeking, and therefore tolerate larger cabinet coalitions.

The bottom right panel of Figure 1 throws further light on this, plotting τ against the probability of a potential Condorcet winning *grand coalition*. For very low values of τ, modeling very risk-seeking or optimistic politicians, the model prediction is almost always a Condorcet winning grand coalition. Since in practice we *very* rarely observe governments that are grand coalitions including every single party in the legislature, we recalibrate τ to a range consistent with a low probability of grand coalitions. In what follows, therefore, informed by the bottom right panel of Figure 1, we confine ourselves to model calibrations in which $\tau > 0.25$.

In contrast, high-tau politicians are very pessimistic or risk averse: they always expect tabling an issue to turn out badly, resulting in the outcome they least prefer. This leads them to dislike large coalitions, which tend to have many tabled issues and, other things equal, to favor single party cabinets which by construction have no tabled issue. While this is again by no means self-evident, we see from the bottom left panel of Figure 1 that, with very high values of τ modeling intensely risk-averse or pessimistic politicians, the model prediction is almost always a Condorcet winning single-party cabinet (typically, though not always, comprising the largest party.) While we do quite often observe single-party minority cabinets, these are by no means the inevitable outcome of government formation in parliamentary systems. In what follows,

therefore, we confine ourselves to model calibrations in which $\tau < 0.75$, consistent with an empirically plausible frequency of single party cabinets.

While increasing the dimensionality of a real policy space (notoriously from one to two), is typically seen to make weighted majority voting equilibria less likely, it may well be that the reverse is true for discrete issues spaces in which policy compromises are reached by logrolling. Trivially, logrolling is impossible in a one-dimensional issue space and is at least possible in two dimensions. Adding further dimensions, all else constant, adds further possibilities for logrolling. The top right panel of Figure 1 shows our logrolling model predicting an increased probability of Condorcet winning potential cabinets as the dimensionality of the discrete issue space increases.

The middle two panels of Figure 1 show that the two noise parameters, σ_p and σ_s, have only a small effect on the probability of Condorcet winning cabinets, though the center right panel is consistent with our observation that, the less correlated the parties' issue saliences, the more scope there is for logrolling to realize gains from trade.[27]

Visualizations aside, the best way to understand the effect of model parameters on model predictions is to investigate them statistically. Table 2 reports multivariate analyses, using logistic regressions, of the effects of model parameters on predictions of Condorcet winning cabinets. Given the strong effects of model parameters, illustrated in Figure 1, on whether predicted Condorcet winning cabinets are single party cabinets or coalitions, we report separate analyses for each type of cabinet, excluding grand coalitions since these are so very

[27] While these effects are "statistically insignificant" as shown (a horizontal line can be drawn through the confidence intervals), as with all such computational estimations increasing the number of repetitions would shrink these confidence intervals.

rare in the real world. This confirms the pattern shown in Figure 1. Condorcet winning single-party cabinets are associated with high-tau politicians and Condorcet winning coalitions with low-tau politicians. Table 2 also refines the aggregate pattern shown in Table 1 in easily interpretable ways. Condorcet winning *single-party* cabinets are predicted most frequently in Type B legislatures, dominated by the largest party. Condorcet winning *coalitions* are predicted most frequently in Type C legislatures, in which any pair of the top three parties is winning.

Table 2: Logistic regressions summarizing the effects of model parameters on predicted Condorcet winning cabinets (0.25 < τ < 0.75, standard errors, in italics, clustered by setting)[28]

Model parameter	Condorcet winning non-grand coalition	Condorcet winning one-party cabinet
m: dimensionality	**0.006** *0.001*	**-0.010** 0.003
τ: aversion to tabled issues	**-7.779** *0.317*	**11.903** *0.722*
Type B: dominant party	**3.641** *0.401*	**3.349** *0.533*
Type C: two three	**4.769** *0.417*	1.591 *0.633*
Type D: top two	**1.883** *0.409*	**2.002** *0.620*
Constant	**-2.823** *0.381*	**-12.733** *0.705*
n	22,786	22,786
pseudo-r^2	0.213	0.240

[28] All coefficients in bold are "statistically significant" at better than the 0.001 level, otherwise at the 0.01 level. Note that these measures of significance are not really relevant for computational work such as this, since we can reduce standard errors to any value we desire simply by increasing the number of repetitions. The relevant information in the table concerns the estimated coefficients.

Sensitive to the possibility that the assumption confining us to bilateral logrolls might cause us to miss something important, we reran our simulations for two countries, computing two additional quantities: the proportion of contested issues that have a single outlier; and the proportion of theoretically possible logrolls (combinations of contested issues) that are identified as logrolls by our behavioral model. We computed these quantities for Denmark, which has a "large and complicated" party system and Germany, with a "smaller and simpler" party system. Table 3 shows the results.

Table 3: Proportion of contested issues with unique outliers and proportion of theoretically possible logrolls identified.

Country	Year	Number of parties	*Prop. contested issues with unique outliers*	*Prop. possible logrolls identified*
Denmark	1988	8	0.61	0.39
Denmark	1990	8	0.61	0.39
Denmark	1994	8	0.60	0.38
Denmark	1998	10	0.41	0.25
Denmark	2001	8	0.59	0.38
Denmark	2005	7	0.71	0.46
Denmark	2007	8	0.60	0.38
Germany	1987	4	0.96	0.63
Germany	1990	5	0.89	0.58
Germany	1994	5	0.89	0.58
Germany	1998	5	0.89	0.58
Germany	2002	5	0.89	0.58
Germany	2005	5	0.90	0.58
Germany	2009	5	0.89	0.58

Unsurprisingly, we find a strong connection between the number of parties in a government formation setting and the number of bilateral trades as a proportion of the total

set of unmatched issues. Particularly in a "small" party system such as that in Germany, a high proportion (around 90 percent) of potential logrolls involve unique outliers, and are therefore covered by our behavioral algorithm. While we believe it would certainly be interesting to explore more complex logrolls, this seems unlikely to change our findings substantially in settings with relatively few parties, like Germany. For systems with relatively many parties, such as Denmark, the combinatorics involved in searching for more complex logrolls in high dimensional issue spaces, given the number of possible coalitions, would require a more complex set of heuristics than those we present here.

However, as we now show, even in large party systems our simple logrolling model adds significant new insight on one of the empirical puzzles that motivates this paper, the well-established fact that "bargaining delays" over government formation are much longer in some settings than in others.

BARGAINING DELAYS

Having mapped out the relative probabilities of Condorcet winning cabinets, conditional on model parameters in different settings, we now use these probabilities to analyze varying bargaining delays over government formation. According to canonical models of government formation, rational politicians with complete information negotiate equilibrium cabinets without delay. We motivated our argument, however, by highlighting the variable and sometimes extended delays typically observed during government formation negotiations in the real world. These delays have attracted the attention of a small number of scholars, who offer explanations relating to uncertainty and/or complexity in the bargaining environment. If the bargaining environment is evolving stochastically, then bargaining delays may arise even

when rational politicians have complete information (Merlo 1997; Merlo and Wilson 1995). Extending this conclusion to negotiations over government formation, Deirmeier and van Roozendaal (1998) found a strong empirical relationship between bargaining delays and uncertainty. These general results were replicated by Martin and Vanberg (2003) and by Golder (2010), who found the strongest indicator of uncertainty to be that government formation negotiations take place immediately after a general election had changed the composition of the legislature, as opposed to "mid-term" negotiations between a continuing set of legislators. The "complexity" of the bargaining environment, measured by Golder and others in terms of the number and ideological diversity of the legislative parties, was also found to affect bargaining delays. More recently, Laver and Benoit (2015) redefined bargaining complexity in terms of the taxonomy of legislative types we deployed above. The least complex bargaining environments are legislatures dominated by a single party and those in which only the top three parties can be pivotal. Laver and Benoit found that, holding constant the number of parties, these are associated with the shortest bargaining delays. The most complex bargaining environments, associated with the longest bargaining delays, are found in legislatures where winning coalitions *must* comprise at least three parties.

We now use our logrolling model to extend these findings by adding a new measure of the "difficulty" of government formation negotiations. Our expectation is that government formation settings with fewer potential Condorcet winning cabinets pose tougher problems for negotiators than those in which it is easier to identify a Condorcet winner. Settings with fewer potential Condorcet winning cabinets are therefore more likely to be associated with bargaining delays. The justification for this claim is straightforward.

Recall that each different potential cabinet is associated with a unique logrolled joint policy position in a high-dimensional issue space. Since the agreed joint program of a proposed government is what matters to legislators, there are many different cabinets associated with the same coalition partners – some Condorcet winners and some not – given the many different policy programs they might negotiate. Government formation negotiations are, other things equal, more taxing when potential Condorcet winning joint policy programs are harder to find in the huge state spaces generated by the high-dimensional issue environment. In contrast, when Condorcet winning joint policy programs are easier to find, we expect bargaining to proceed more rapidly, not least because these proposals, once identified, are proof against any alternative. An additional way of thinking about this concerns the ordering of the issue agenda, which as noted we randomize over repeated trials in our computational work. *Within a given protocoalition*, we have argued that we can interpret each ordering as a "negotiation attempt". The smaller the probability a negotiation attempt results in a Condorcet winning logroll, the more negotiation attempts are required; assuming each negotiation attempt takes time, the longer government formation should take.

Having computed relative frequencies of model-predicted Condorcet winners for each government formation setting and a range of parameter values calibrated as we described above, we use the relative difficulty of identifying a Condorcet winner according to our model to predict observed bargaining delays. Extending results reported by Laver and Benoit, we also

model delays as a function of: absolute and effective number of parties;[29] post electoral setting; and legislative type. We computed the legislative type ourselves and took other data from the *European Representative Democracy* (ERD) dataset.[30] Our dependent variable is the number of days between the holding of an election or the resignation of an incumbent government during an inter-electoral period and the formation of a new government according to the ERD dataset.[31] The observed distribution of these bargaining delays is close to Poisson and Table 4 replicates earlier published work by reporting Poisson regressions predicting the duration of government formation negotiations using these variables.[32] The first column reports results for the complete set of calibrated cases.[33] Given the small number of observed cases, as a robustness check the second column shows re-estimates standard errors using a 1000-replication suite of bootstrapped Poisson regressions. Table 4 replicates the findings of previous studies that, holding constant the number of parties, shorter bargaining delays tend to be associated with "mid-term" government formations and with legislatures dominated by the largest party.

[29] The absolute number of parties determines the number of different possible coalitions. For a given absolute number of parties, a higher effective number of parties indicates a more fragmented party system, and other things equal a more "difficult" government formation process.

[30] www.erdda.se

[31] This period may have included several formation attempts, which may or may not have been documented in contemporary sources, but our central concern here is with how long it took to form a government in each setting, not in how many failed attempts there might have been.

[32] Martin and Vanberg (2003), Golder (2010), Laver and Benoit (2015)

[33] Two of our calibrated government formation setting were not included in the ERD data, leaving 89 in all.

Table 4: Poisson regressions predicting bargaining delays from relative frequency of predicted Condorcet winners, effective number of parties, post-electoral setting, and legislative typem (standard errors in italics)

Model parameter	All cases	Bootstrap (1000 reps)
Probability of Condorcet winner	**-3.871** *0.264*	**-3.871** 1.603
Post electoral setting	**2.259** *0.103*	**2.259** *0.383*
Absolute number of parties	0.001 *0.024*	0.001 *0.179*
Effective number of parties	**-0.363** *0.041*	-0.363 *0.390*
Type B: dominant party	**-0.672** *0.059*	**-0.672** *0.295*
Type C: top three	**0.236** *0.071*	0.236 *0.466*
Type D: top two	**-0.467** *0.059*	-0.467 *-0.381*
Constant	**4.085** *0.222*	**4.085** 1.463
n	89	89
pseudo-r^2	0.362	0.362

Returning to the original motivation of our argument, Table 4 also supports the claim that settings with fewer potential Condorcet winning cabinets make government formation more difficult and are associated with longer bargaining delays. *This is true holding constant legislative type, number of parties and post-electoral setting.* Table 4 therefore shows that the probability of a potential Condorcet winning cabinet, computed according to our model and interpreted as a measure of the number of negotiation attempts needed to find an agreed

program of government that cannot be beaten in the legislature, adds significantly to our ability to predict bargaining delays, over and above previously published results.

We also use the lasso to identify the variables most closely associated with bargaining delays.[34] Figure 2 shows the relative contribution of each independent variable to model fit as the lasso regularization penalty is loosened. On the far left of the figure, the penalty is very high and all independent variables, by design, have slopes of zero and effectively fall out of the model. On the far right, the penalty is very low and all independent variables, save the absolute number of parties, enter the model with a non-zero slope. The interior allows us to assess the relative contribution of each independent variable to explaining bargaining delays, based on the fraction of the population deviance explained as the lasso penalty is reduced. We see that by far the strongest predictor is our model's estimated probability of Condorcet winners, which we interpret as a measure of the difficulty of finding and/or negotiating an equilibrium cabinet. This is followed by the post electoral setting, identified by previous authors as an important predictor of bargaining delays. The other independent variables in Table 4 (effective number of parties and legislative type) are not the main drivers of bargaining delays and only contribute to the model for low levels of the regularization penalty.

Figure 2 suggests that our model contributes to the understanding of the variable bargaining delays which motivated this paper. As we discussed above, previous authors identified the absolute or effective number of parties, and legislative type, as being associated

[34] The lasso is a machine learning algorithm for variable selection that shrinks the model by recursively penalizing the absolute size of regression coefficients until those associated with the least influential variables are driven to zero.

in a general way with the "difficulty" of government formation negotiations in a particular setting. Building on these findings, our model provides a much more direct measure of the difficulty of such negotiations. This is the likelihood, in any given setting, of potential Condorcet winning cabinets arising from logrolling. The harder it is to find a set of logrolls which generate an agreed government policy program that beats all others in pairwise comparisons, the longer the bargaining delay. This strongly suggests that logrolling over policy outcomes is an important feature of government formation in parliamentary systems.

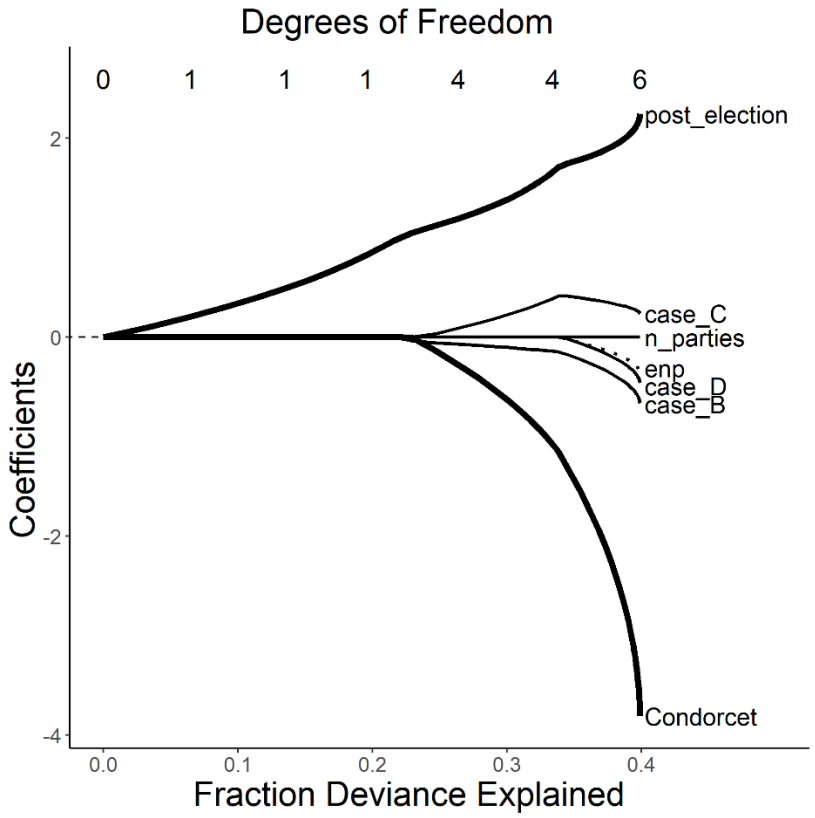


Figure 2: Lasso plot showing fraction of deviance explained by the independent variables used to explain bargaining delays.

CONCLUSIONS AND FUTURE WORK

Our core argument is that we increase our understanding of government formation in parliamentary democracies by modeling this as logrolling in high dimensional issue spaces. We set up this approach with some very general assumptions: high dimensionality; heterogeneous party-specific issue saliences; the possibility of tabling issues on which negotiators agree to disagree. A wide variety of logrolling models are possible within this general rubric. We narrow things down enough to build a particular model of logrolling with more specific assumptions: binary issue positions; non-strategic behavior; simple bilateral trades. The particular model we specify and analyze is stochastic, reflecting the fundamental uncertainty in which government formation negotiations take place. We exercise our model here to accomplish two things.

First, we map out the probability of Condorcet winning potential cabinets in real settings, conditional on model parameters. We show that potential Condorcet winning cabinets are not only possible, but that these *can be identified by negotiating politicians using our proposed behavioral algorithm of simple bilateral logrolling.* Bilateral logrolling, in short, can identify potential Condorcet winning cabinets in the high dimensional issue spaces which we argue self-evidently characterize government formation in the real world.

Second, we extend existing research on why government formation takes much longer in some settings than others. Theoretically, perfectly rational politicians with complete information negotiate equilibrium cabinets without delay. This is why existing work on bargaining delays has focused on empirical measures of the "difficulty" of government formation negotiations in complex environments with incomplete information. Our contribution is to argue that the setting for government formation is a high-dimensional issue

space, which poses huge cognitive and informational challenges to negotiators. We claim that negotiating politicians in such settings navigate the resulting gigantic state space using behavioral algorithms rather than hyper-rational strategic analysis. We propose one such algorithm and deploy our model to show that some government formation settings are much more "difficult" than others for negotiators, in the sense that logrolling generates far fewer potential Condorcet winners. This means that more "negotiation attempts" are required before negotiating politicians find a Condorcet winner. Since each negotiation attempt takes time, this implies longer bargaining delays in such settings. Our empirical tests in Table 4 and Figure 2 provide strong empirical support for our computational model.

It is clear from Assumptions 1 – 3 that ours is far from being the only possible model of "government formation as logrolling". What we have shown here is that even the very simple model we propose seems to have significant explanatory power. There are many possible ways to build on this. More complex Pareto-improving logrolls involving more than two parties and more than two issues are clearly possible, though we have shown the scope for these is much greater in large party systems than in small ones. Some issues may be intrinsically continuous and not admit a binary representation. More sophisticated behaviors than the greedy algorithm we propose are clearly possible. Issue preferences, and especially saliences, may be intrinsically private information. Logrolling may be complicated by the possibility of side payments, trading perks such as cabinet portfolios, for example, for policy concessions. It is not hard to think of ways to complicate the basic idea set out in this paper. Our claim is that even our simple baseline model shows that the idea of government formation as logrolling is worth developing.

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References

Aksoy, Deniz. 2012. "Institutional Arrangements and Logrolling: Evidence from the European Union." *American Journal of Political Science* 56 (3): 538-52.

Andersson, Staffan & Svante Ersson. 2012. "The European Representative Democracy Data Archive". Main sponsor: Riksbankens Jubileumsfond (In2007-0149:1-E). Principal investigator: Torbjörn Bergman. www.erdda.se

Ansolabehere, Stephen, James M. Snyder, Aaron B. Strauss, and Michael M. Ting. 2005. "Voting weights and formateur advantages in the formation of coalition governents." *American Journal of Political Science* 49 (3):550-563.

Austen-Smith, D., & Banks, J. .1990. "Stable governments and the allocation of policy portfolios." *American Political Science Review 84* (3), 891-906.

Axelrod, Robert. 1970. *Conflict of Interest*. Chicago: Markham.

Banzhaf, J. F. III. 1965. "Weighted voting doesn't work: a mathematical analysis." *Rutgers Law Review* 19 (2): 317-343.

Baron, David and John A. Ferejohn. 1989. "Bargaining in legislatures." *American Political Science Review* 83 (4): 1182-1206.

Baron, David. 1991. "A spatial bargaining theory of government formation in parliamentary systems." *American Political Science Review* 85 (1), 137-165.

Baron, David P., and Daniel Diermeier. 2001. "Elections, governments, and parliaments in proportional representation systems." *The Quarterly Journal of Economics* 116 (3): 933-967.

Benoit, Kenneth, and Michael Laver. 2006. *Party policy in modern democracies*. Routledge.

Bowler, Sean, Indridi Indridason, Tomas Bräuninger, and Marc Debus. 2016. "Let's just agree to disagree: dispute resolution mechanisms in coalition agreements." *The Journal of Politics 78* (4): 1264-1278.

Browne, Eric C. and Mark Franklin. 1973. "Aspects of coalition payoffs in European parliamentary democracies." *American Political Science Review* 67 (2): 453-469.

Carrubba, Clifford J, and Craig Volden. 2000. "Coalitional Politics and Logrolling in Legislative Institutions." *American Journal of Political Science* 44 (2): 261-77.

Converse, Phillip E. 1964. "The Nature of Belief Systems in Mass Publics." In *Ideology and Discontent.* , ed. David E. Apter. London: Free Press of Glencoe. 206-61.

De Swaan, Abram. 1973. *Coalition Theories and Cabinet Formation*. Amsterdam: Elsevier.

Diermeier, Daniel, and Peter Van Roozendaal. 1998. "The duration of cabinet formation processes in western multi-party democracies." *British Journal of Political Science* 28 (4): 609-26

Dodd, Lawrence C.. 1976. *Coalitions in Parliamentary Government.* Princeton: Princeton University Press.

Duggan, John. 2000. "A bargaining model of collective choice." *American Political Science Review* 94(1): 73-88.

- Ecker, Alejandro, and Thomas M. Meyer. 2017. "Coalition Bargaining Duration in Multiparty Democracies." *British Journal of Political Science*: 1-20 https://doi.org/10.1017/S0007123417000539

Einy, Ezra, 1985. "On connected coalitions in dominated simple games." *International Journal of Game Theory* 14(2): 103-25

Falcó-Gimeno, A. 2014. "The use of control mechanisms in coalition governments: The role of preference tangentiality and repeated interactions." *Party Politics* 20(3) 341-356.

Gamson, William A. 1961. "A theory of coalition formation." *American Sociological Review* 26 (3): 373-382.

Golder, Sona N. 2010. "Bargaining delays in the government formation process." *Comparative Political Studies* 43 (1):3–32.

Golder, Matt, Sona N. Golder, and David A. Siegel. 2012 "Modeling the institutional foundation of parliamentary government formation." *The Journal of Politics* 74(2): 427-445.

Grofman, Bernard. 1982. "A dynamic model of protocoalition formation in ideological n-space." *Behavioural Science* 27 (1), 77-90.

Hortala-Vallve, Rafael. 2011. "Generous legislators? A description of vote trading agreements." *Quarterly Journal of Political Science 6*(2), 179-196.

Humphreys, Macartan. 2008. "Coalitions." *Annual Review of Political Science* 11: 351-86.

Krasa, Stefan and Mattias Polborn. 2010. "The binary policy model." *Journal of Economic Theory* 145 (2), 661-688,

Laver, Michael and William Ben Hunt. 1992. *Policy and party competition*. Routledge.

Laver, Michael, Scott de Marchi, and Hande Mutlu. 2011. "Negotiation in legislatures over government formation." *Public Choice* 147(3-4): 285-304.

Laver, Michael, and Kenneth Shepsle. 1990. "Coalitions and cabinet government." *American Political Science Review 84*(3), 873-890.

Laver, Michael and Kenneth A. Shepsle. 1996. *Making and breaking governments.* New York: Cambridge University Press.

Laver, Michael, and Kenneth Benoit. 2015. "The basic arithmetic of legislative decisions." *American Journal of Political Science 59*(2), 275-291.

Leiserson, Michael A. 1966. *Coalitions in Politics: A Theoretical and Empirical Study*. New Haven: Unpublished PhD thesis, Yale University.

Luebbert, Gregory. 1986. *Comparative Democracy: Policy Making and Government Coalitions in Europe and Israel.* New York: Columbia University Press.

Martin, Lanny W., and Georg Vanberg. 2003. "Wasting Time? The Impact of Ideology and Size on Delay in Coalition Formation." *British Journal of Political Science* 33 (2): 323-32.

Martin, Lanny W., and Georg Vanberg. 2014 "A step in the wrong direction: An appraisal of the zero-intelligence model of government formation." *The Journal of Politics* 76(4): 873-879.

Martin, Lanny W, and Georg Vanberg. 2015. "Coalition Formation and Policymaking in Parliamentary Democracies." *Routledge Handbook of Comparative Political Institutions*.

McKelvey, Richard D. and Norman Schofield. 1986. "Structural instability of the core." *Journal of Mathematical Economics* 15 (3): 179-198.

McKelvey, Richard D. and Norman Schofield. 1987. "Generalized symmetry conditions at a core point." *Econometrica* 55 (4): 923-933.

Merlo, Antonio. 1997. "Bargaining over governments in a stochastic environment." *Journal of Political Economy* 105 (1):101-31.

Merlo, Antonio, and Charles Wilson. 1995. "A stochastic model of sequential bargaining with complete information." *Econometrica* 63 (2):371-99.

Morelli, Massimo. 1999. "Demand competition and policy compromise in legislative bargaining." *American Political Science Review 93*(4), 809-820.

Penn, Elizabeth Maggie. 2006. "Alternate definitions of the uncovered set and their implications." *Social Choice and Welfare* 27(1): 83-87.

Quinn, Thomas, Judith Bara and John Bartle. 2011. "The UK Coalition Agreement of 2010: Who Won?" *Journal of Elections, Public Opinion and Parties,* 21(2): 295-312

Riker, William H. 1962. *The theory of political coalitions*. Yale University Press.

Rubenstein, Ariel. 1982. "Perfect equilibrium in a bargaining model." *Econometrica* 50(1): 97-109.

Schofield, Norman and Michael Laver. 1985. "Bargaining theory and portfolio payoffs in European government coalitions." *British Journal of Political Science.* 15(2): 143-164

Shepsle, Ken A, and Torun Dewan. 2008. "Recent Economic Perspectives on Political Economy, Part 1." *British Journal of Political Science* 38(2): 362 - 82.

Stokes, Donald E. 1963. "Spatial Models of Party Competition." *American political science review* 57 (2): 368-77.

Stratmann, Thomas. 1992. "The Effects of Logrolling on Congressional Voting." *The American Economic Review* 82 (5) 1162-76.

Stratmann, Thomas. 2004. "Logrolling 1." *The Encyclopedia of Public Choice*: Springer. 696-99.

Strom, Kaare. 1990. "A Behavioral Theory of Competitive Political Parties." *American Journal of Political Science* 34 (2): 565-98.

APPENDIX A

Table A1: Government formation settings calibrated[35]

Austria	Election	1986	1990	1994	1995(2)	1999	2002	2006
	N parties	4	4	5	5	4	4	5
	Leg type	C	C	C	C	C	B	D
	Survey	LH	LH	LH	LH	BL	BL	CH06
Denmark	Election	1988	1990(2)	1994(2)	1998	2001	2005	2007(2)
	N parties	8	8	8	10	8	7	8
	Leg type	D	B	B	D	D	D	D
	Survey	LH	LH	LH	BL	BL	CH06	CH06
Finland	Election	1987(2)	1991(2)	1995	1999(2)	2003(2)	2007(2)	2011
	N parties	9	9	10	8	8	8	8
	Leg type	D	D	B	D	D	D	E
	Survey	LH	LH	LH	BL	BL	CH06	CH10
Germany	Election	1987	1990	1994	1998	2002	2005	2009
	N parties	4	5	5	5	5	5	5
	Leg type	B	B	B	B	C	D	B
	Survey	LH	LH	LH	BL	BL	CH06	CH10
Iceland	Election	1987(3)	1991	1995	1999	2003(3)	2007(2)	2009
	N parties	7	5	6	5	5	5	5
	Leg type	E	B	B	B	C	B	B
	Survey	LH	LH	LH	BL	BL	CH06	CH10
Ireland	Election	1987	1989(2)	1992(2)	1997		2007	
	N parties	7	8	7	9		7	
	Leg type	B	B	B	B		B	
	Survey	LH	LH	LH	BL		CH06	
Luxembourg	Election	1989	1994(2)	1999	2004	2009		
	N parties	7	5	6	5	6		
	Leg type	B	B	B	B	B		
	Survey	LH	LH	BL	CH06	CH10		

[35] The table lists relevant general elections in each country during the period 1986-2010. Each cell defines a calibrated government formation "setting" for our model, and identifies the closest expert survey estimate to the time of the election. LH = Laver-Hunt; BL = Benoit-Laver; CH06 and CH10 are the 2006 and 2010 Chapel Hill Expert surveys. Figures in parenthesis after an election year indicate the number of multiple government formations before the following election.

Netherlands	Election	1989		1998	2002	2003(2)		2010
	N parties	9		9	10	9		10
	Leg type	B		D	E	D		E
	Survey	LH		BL	BL	BL		CH10
Norway	Election	1989(2)	1993(2)	1997(2)	2001	2005	2009	
	N parties	7	8	8	8	7	7	
	Leg type	B	B	B	E	D	B	
	Survey	LH	LH	BL	BL	CH06	CH10	
Portugal	Election			1995	1999	2002(2)		2009
	N parties			5	6	6		5
	Leg type			B	B	B		B
	Survey			LH	BL	BL		CH10
Sweden	Election	1988(2)	1991	1994(2)	1998	2002	2006	2010
	N parties	6	7	7	7	7	7	7
	Leg type	B	D	B	D	B	D	D
	Survey	LH	LH	LH	BL	BL	CHO6	CH10

APPENDIX B: Simulating distributions of high-dimensional discrete issue spaces

Our objective is to generate for each trial a distribution of *m*-dimensional discrete issue spaces, each consistent with an underlying empirical setting that is described by a δ-dimensional real space in which party positions on any dimension are measured on the [0,1] interval and each party's relative saliences for all dimensions sum to unity. For each of a specified number, *r*, of model runs we first randomly perturb data describing agents' policy positions and saliences in the underlying low-dimensional context. We then round agents' perturbed positions to binary numbers, and rescale perturbed saliences to sum to unity. This generates a distribution of *r* high dimensional discrete issue spaces, each derived from the underlying context, in a process analogous to the bootstrap.

We do this as follows for each underlying empirical setting. For each underlying real dimension *d*, we generate μ_d discrete realizations of this. The total number of issue dimensions generated for each discrete space is $m = \Sigma_d \mu_d$. To generate agent *i*'s position, p_{ij} on discrete issue dimension *j* from its position on underlying real policy dimension *d*, we perturb *i*'s observed position p_{id} ($\in$ [0,1]) on *d*: $p'_{id} = p_{id} + \varepsilon$, where ε is Gaussian noise with mean 0 and standard deviation σ_p. We then round p'_{id} to 0 or 1, generating *i*'s binary position p_{ij}. In effect σ_p is a tunable "issue noise" parameter specifying how closely parties' typical high-dimensional issue positions in each policy area refer to the underlying low-dimensional context. Thus $\sigma_p = 0$ yields a binary representation of the underlying policy context, but every one of *r* replications of this process will be identical. The higher σ_p, the closer the high-dimensional issue position to a random binary number, and the more different from each other each of the *r* replications of this process will be. The underlying party issue saliences are perturbed in directly analogous

way, using a "saliency noise" parameter σ_s. The perturbed saliences are not rounded to integers, but linearly rescaled to sum to unity across all issue dimensions for each party.

We replicate this process *r* times to generate a distribution of *r* discrete *m*-dimensional issue spaces, all derived from the same underlying context. We can think of the distribution of discrete issue spaces derived from a given empirical setting in two ways. One is a pragmatic response to the fact we have good data on the latent low-dimensional structure of the manifest high-dimensional issue space, but not on the manifest issue space itself. We thus generate a distribution of plausible manifest realizations and work with these. A deeper interpretation is to think in terms of fundamental uncertainty. The distribution of manifest issue spaces is a set of potential realities consistent with the same latent structure, while the issue space which is actually realized is simply a random draw from this. In a stochastic universe, random events might easily have served us a different draw from this distribution.