Negotiation in legislatures over government formation*

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ABSTRACT

We question results claiming to extend non-cooperative models of legislative bargaining to the theoretically general and substantively typical case with an arbitrary number of disciplined political parties. We identify problems with both derivation of theoretical results and empirical evaluation of these. No empirically robust *formateur* advantage can be observed in field data on bargaining over government formation. Given this theoretical and empirical *impasse*, we reconsider the substantive premises that should form the foundation for any new attempt to model this fundamental political process, arguing these premises should be grounded in binding constitutional constraints on the government formation process in parliamentary democracies.

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1. INTRODUCTION

Bargaining in legislatures may involve the division of private goods, the setting of public policy, and many other things besides. It is of special importance in parliamentary democracies, where the most important decisions facing legislators concern the life and death of governments. Theorists typically assume politicians engaged in such bargaining lead disciplined political parties, making important legislative decisions by majority voting. The substantive setting analyzed is thus of weighted voting under majority rule, in a legislature defined as a partition of m legislators into n political parties. The number of legislators, l_i, belonging to party i gives the party its "raw" voting weight.[1] Recent work on bargaining in legislatures focuses with extraordinary consistency on building out from an alternating-offers bargaining foundation laid down by Rubenstein and exploited by Baron and Ferejohn, hereafter BF (Baron and Ferejohn 1989; Rubenstein 1982). Subsequent models claiming BF ancestry retain core features of the original model but use different parameter settings and institutional assumptions, generating a diverse family of BF-style bargaining models.

The common pedigree of BF-style bargaining models includes: an exogenously determined and fixed set of disciplined political parties; an exogenous automaton that determines a vector $\boldsymbol{L}$ specifying the number of legislators, l_i controlled by each party i; and an exogenously determined quota Q of legislators required to pass any proposal. There is a random recognition automaton, parameterized by a vector $\boldsymbol{P}$ of exogenously determined common knowledge recognition probabilities, p_i, for each party. At each stage in the bargaining process, a single agent is picked by the automaton to have the monopoly right to make one proposal to other agents. Agents not recognized by the automaton may not make any proposal and proposals are immediately voted on without discussion. All legislators vote and, if votes in favor equal or exceed Q, the proposal is instantly implemented.[2] The game then ends immediately and all payoffs are consumed. If a proposal does not pass the winning threshold, another monopoly proposer is selected by the automaton. If no agreement is reached, payoffs to all actors are typically normalized to zero. The most distinctive of these assumptions is the exogenous mechanism for selecting the proposer, or *formateur,* with monopoly power to make proposals at each point in time. Due in large part to this assumption, the core prediction of most BF-style bargaining models is that, in equilibrium, there is a minimum winning coalition (MWC) of agents, in which other members receive their continuation values in the bargaining game and the *formateur* retains the balance.

BF themselves (1989: 1193) suggest "government formation in parliamentary systems" as an application of their model, though "for simplicity" illustrate this with a three-party legislature with no majority party. But analyzing this simple setting avoids many of the

[1] We use the term "raw" voting weight here to distinguish the substantive fact of the number of legislative votes under the control of each party leader from (often quite different) theoretically inspired notions of voting weight we return to below.

[2] In real parliamentary democracies, precise constitutional rules, clearly part of the political game, specify institutional procedures for implementing such proposals. In BF-style models, agreements between parties are implicitly assumed to be exogenously enforced; once they are made, the game ends and agreements are automatically and perfectly implemented.

complications that must be resolved when analyzing weighted voting under majority rule. In a three-party minority legislature, and more generally in a perfectly symmetrical *n*-party setting in which any party can be substituted for any other party in any MWC, it is not necessary to resolve issues arising from "dummy" parties that are members of no MWC, or from "non-homogenous" weighted voting games. An example of the latter is a five party 14-seat legislature with partition of seats between parties of (4, 3, 3, 2, 2) and 8 votes needed to pass a proposition. Consider the MWCs (4, 3, 3) and (3, 3, 2). An excluded 2-seat party is "substitutable" for a 3-seat party in the first MWC – switching parties leaves this coalition winning – but not in the second MWC.[3] This game is "non-homogenous" in the precise sense that, describing a weighted voting game in terms of "minimum integer weights" (MIWs), not all MWCs have the same aggregate MIW.[4] The generic theoretical issue arising in non-homogenous games is that two parties may be substitutes for each other in some MWCs but not in others.

Extension of bargaining models to *n*-party legislatures with arbitrary voting weights

Snyder, Ting and Ansolabehere (hereafter STA) set out to extend the BF alternating offers approach to deal with non-homogenous voting games and other complications arising in arbitrary *n*-party settings with weighted voting (Ansolabehere et al. 2005; Snyder, Ting, and Ansolabehere 2005). They develop a theoretical model of *n*-party legislative bargaining with arbitrary voting weights and claim empirical support in field data on the distribution of cabinet seats between parties, following bargaining over government formation. STA (2005: 982) apply what they describe as a basic insight of "elementary microeconomic theory", which:

> … teaches that in competitive situations perfect substitutes have the same price. In a political setting in which votes might be traded or transferred in the formation of coalitions, one might expect the same logic to apply. If a player has *k* votes, then that player should command a price for those votes equal to the total price of *k* players that each have one vote. In terms of expected payoffs, the player with *k* votes should expect to have a payoff *k* times as great as the payoff expected by a player with one vote.

Building on this argument, the core result generated by STA can be found in their Propositions 2 and 3, stating that agents' continuation values in an *n*-party BF-style bargaining game are proportional to their voting weights.[5] The implication is that *formateurs*, once selected by the

[3] The first replacement leaves the winning coalition (4, 3, 2), the second the losing (3, 2, 2).

[4] The vector $\boldsymbol{M}$ of MIWs, m_i, for each party i is defined as the smallest set of integers that generates, for a given winning quota Q, the same set of winning coalitions $\boldsymbol{C}$ as does the raw seat vector $\boldsymbol{L}$. A (dummy) party that is an essential member of no winning coalition has an MIW of zero. Montero (2006) gives a rigorous formal definition of non-homogenous games.

[5] This proposition is somewhat more complex than stated here. STA (p 986) claim that "the price a type-*t* coalition partner can command equals that player's continuation value … divided by his or her share of the voting weight in the *r*th replication". The intuition, however, is the same and we discuss the crucial role of STA "replication" approach in Section 3. STA's Proposition 4 modifies this conclusion somewhat for certain corner solutions when recognition probabilities are equal for all agents, but equilibrium continuation values remain monotonic in voting weights.

random automaton, offer coalition partners in some MWC their continuation values, retaining the surplus and resulting once more in a *formateur* advantage. If correct, these results would be very significant, extending the BF non-cooperative bargaining model to the more general context of *n* disciplined political parties with arbitrary voting weights, raising the possibility, among other complications, of non-homogenous games.[6]

The Puzzle of Gamson's Law

Very striking in this particular substantive context is a strong empirical regularity, Gamson's Law (GL). In the real world, the proportion of cabinet ministries received by each government party, following bargaining over the distribution of these, tends strongly to equal the proportion of legislative seats contributed by that party to the government seat total. This strong empirical regularity contradicts canonical bargaining models in predicting *no formateur advantage*. GL has been tested and retested many times and is remarkably robust (Browne and Franklin 1973; Browne and Frendreis 1980; Gamson 1961; Laver and Schofield 1998; Warwick and Druckman 2001; Warwick and Druckman 2006). This generates what Warwick and Druckman (2006) call the "portfolio allocation paradox". The profession's canonical theory of bargaining in legislatures is contradicted by one of the profession's strongest and most robust empirical laws.

Figure 1 illustrates this paradox, using STA's replication dataset to plot government parties' portfolio payoffs against their shares of the government's legislative seat total. GL states that parties' portfolio payoffs are proportional to their shares of the government's legislative seat total; this can be seen clearly in Figure 1. BF-style bargaining models predict that *formateur* parties (plotted "x") *will systematically receive a higher share of the portfolio payoffs* than non-*formateur* parties (plotted "o") that make the same contribution to the government's seat total.[7] Figure 1 contradicts this. Larger parties tend to get more; *formateur* parties tend to get more only because they tend to be larger. The middle part of the scatterplot shows that *formateur and non-formateur parties of the same size tend strongly to get the same payoffs.* Figure 1 does not suggest "a strong, significant, formateur advantage … consistent with proposal-based bargaining models" (Ansolabehere et al. 2005: 561).

We address this problem as follows. We investigate empirical claims about BF-style bargaining models in Section 2, finding support for these to disappear once account is taken of the fact that the crucial independent variable, *formateur* status, is (and must be) endogenously coded in the data. This leads us in Section 3 to re-evaluate STA's core propositions about the general *n*-party case. We find problems with a proof strategy used to deal with non-homogenous weighted voting games, but applied to all results. Given these theoretical and empirical problems, we set out in Section 4 to lay foundations for more realistic models of bargaining and negotiation between party leaders over government formation, models premised on formal and binding constitutional constraints on government formation in all parliamentary democracies.

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[6] Authors' calculations based in the STA replication dataset show that over one-third of all legislatures analyzed by STA generate non-homogenous games.

[7] BF style models predict that, other things equal, *formateur* will get more; they thus predict that, of two parties with the same share of the government seat total, the *formateur* will get more.

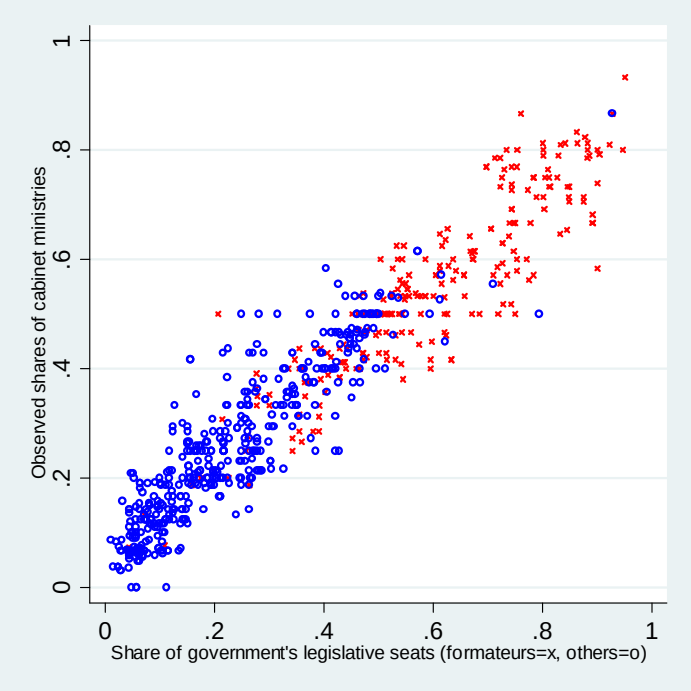


Figure 1: Observed party shares of cabinet ministries, by observed shares of legislative seat total controlled by government parties. Formateur parties x*, non-formateur parties* o*)*
(Source STA replication dataset)

2. NON-ROBUSTNESS OF *FORMATEUR* BONUS IN FIELD DATA

As noted above, STA test model predictions using field data on portfolio allocations in coalition cabinets. Regressing portfolio allocation shares on voting weight shares and adding a dummy variable for observed *formateur* status, they infer empirical support for their model from the fact that the coefficient for the formateur dummy is positive and statistically significant, providing "strong evidence that the parties chosen to form a coalition typically receive more than their voting weight" (STA: 994). It is hard to escape the empirical conclusion, however, illustrated in Figure 1, that there is no systematic *formateur* bonus; that *formateur* and non-*formateur* parties of the same size tend to get the same portfolio payoffs; and that the apparent *formateur* bonus arises because *formateur* parties tend to be larger. The problem is complicated by the fact STA's empirical analysis changes traditional empirical work on Gamson's Law in two ways at the same time. The first change is to introduce a *formateur* variable into predictions of portfolio shares; this is the seminal BF extension; a significant positive coefficient on this variable is held to vindicate a *formateur* model. The second change is to substitute parties' "theoretical voting weights", specifically MIWs, for their raw seat shares. This is a new departure and not a feature of the original BF model as applied to government formation, which uses raw seat shares and sets recognition probabilities proportional to these.[8] Indeed, if parties' raw seat shares are used

[8] Diermeier and Merlo (2004) find empirical recognition probabilities tend to be proportional to raw seat shares.

rather than MIWs, then STA report in a footnote (fn23) that there is no *formateur* bonus and the classic Gamson's Law regressions hold.

We thus identify two distinct empirical claims by STA: (i) MIWs should be used rather than raw seat shares when predicting portfolio payoffs (ii) conditional on (i) being true, there is a *formateur* bonus (STA: 993). Warwick and Druckman (2006) found the following in relation to these claims. In relation to claim (i), raw seat shares are *better* than MIWs in predicting portfolio allocations, when both are included in the same statistical analysis; STA left this as "a matter for future study" (Ansolabehere et al. 2005: 558). In relation to claim (ii), and in line with Figure 1, the *formateur* bonus predicted by BF-style models largely disappears after controlling for the fact *formateur* parties tend strongly to be larger than non-*formateur* parties.[9] As we now show, however, empirical tests of BF-style bargaining models face far a more serious problem than this, causing us to question fundamental modeling assumptions.

Endogenous *formateur* coding

As we have seen, the grounding institutional assumption of alternating offers bargaining models is an exogenous automaton that first selects a *formateur* and then reveals this as common knowledge to all agents. We rarely observe this revelation in the real world, but empirical analyses of BF-style bargaining over government formation fundamentally require coding "*formateur* status" of each political party, observed at the start of the bargaining process. The codings of *formateur* status that underpin STA's empirical work were supplied by Warwick (Ansolabehere et al., 2005: 556). Consulting Warwick and Druckman (2001: 634), we see that *formateur* status was coded from *Keesing's Contemporary Archives*. The following entry in *Keesing's* describes the formation of a German government in 2005. Crucially, this deals with events leading up to, but not including, the eventual formation of a government. It is thus a description of legislative bargaining, taken from the primary source in this field, but one that *does not use the benefit of hindsight* about the eventual outcome of the process under analysis:

> After the results were declared, Schröder controversially claimed that he was the victor because the SPD remained the largest single party, discounting the fact that the CDU and the CSU formed a single group in the Bundestag. Merkel responded that, as the leader of the largest parliamentary group, she had the right to head a new government. However, talks between her and the Greens on Sept. 23 on the formation of a "Jamaica" majority coalition – named after the black (CDU/CSU), yellow (FDP), and green colors of the Jamaican flag – quickly failed. At the same time, the FDP maintained its refusal to enter a "traffic light" coalition with the SPD and the Greens. The only viable option for a majority government, therefore, was a "grand coalition" of the CDU/CSU and the SPD, although at end-September Merkel and Schröder were both still insisting that they should be Chancellor.[10]

[9] Warwick and Druckman also measure the salience of different cabinet portfolios and find portfolio payoffs remain proportional to legislative seat shares, taking account of the possibility some portfolios are worth more than others.
[10] *Keesing's Record of World Events*, Vol.51, 2005 (September) – Europe - Germany

Who, on this basis, should be coded as exogenously determined common knowledge *formateur*? The answer is far from clear and the problem is generic. *Keesing's* almost never contains statements of the form "… after the September election in X, the *formateur* was Y". Primary sources contain discursive accounts of contemporary events such as the one quoted above. These discursive accounts must be read by a human expert who then generates a binary variable for each party by coding its *formateur* status.

Table 1 is generated from the STA replication dataset using the same case universe as their published results. It shows the relationship between a party's coded *formateur* status and whether or not it held the position of Prime Minister (PM) in the eventual government. The pattern is as startling as any we ever see in the social sciences, strongly suggesting that *row and column variables measure precisely the same thing*. This raises the possibility that *formateur* status was coded, not as an *exogenous* independent variable but, *endogenously*, on the basis of whether or not the party took the PM position at the end of the government formation process. "Exogenously" determined *formateur* status and the endogenous control over the PM position, while theoretically distinct, are observationally identical in these data. While no written coding protocol survives, personal communication with Warwick confirmed that eventual control of the PM position was the default criterion for coding *formateur* status, which explains the remarkable pattern observed in Table 1.

Table1: Formateur status and eventual control of PM position[11]

	Party controls eventual PM?		*Total*
	No	Yes	
Party is:			
Non-*formateur?*	1369	0	1369
Formateur?	1	249	250
Total	1370	249	1619

Data source: Replication dataset for STA.

This has two crucial consequences. First, at a methodological level, the key "independent" variable in this dataset – and this is the main dataset used to evaluate BF-style bargaining models using field data – was endogenously coded in light of the very effect it is claimed to predict. This negates the validity of any causal inference drawn from the empirical findings and means the same variable appears on both sides of regression equations estimated both by STA and by Warwick and Druckman (2006). *Formateur* status is the model's key independent

[11] The single off-diagonal case arises from the Ciampi 1 government, forming in Italy in 1993, where the PM is described as a "technician". The number of cases is larger than that in STA's published regressions because the regressions include only parties in government, while Table 2 includes all parties in the relevant legislatures.

variable; the very same thing, in the guise of control over the PM position, is part of the dependent variable, share of cabinet positions. The effects of doing this are exaggerated when, as in some of the STA (2005: 933) regressions, the impact of the PM position on the dependent variable is weighed three times more highly than any other cabinet post.

Table 2 replicates (in models A-C) core empirical results published by STA and Warwick and Druckman (2006), and then (in models D-F) corrects the endogeneity problem in models A-C by subtracting the PM position from the dependent variable. Model A perfectly retrieves STA's published result (Ansolabehere et al., 2005: 557). The significant positive coefficient on the *formateur* dummy is used by STA to infer BF-style formateur models are effective at predicting portfolio payoffs.

Table 2: Portfolio shares, voting weights, formateur status and legislative seat shares

	A: STA Table 3	***B: All govts***	***C: ≤5 parties***	***D: STA Table 3***	***E: All govts***	***F: ≤5 parties***
Dependent variable	*Party proportion of cabinet portfolios*			*As models A-C minus PM*		
Formateur status	0.15**	0.07**	0.09	0.08	0.00	0.00
	(0.05)	(0.02)	(0.04)	(0.05)	(0.02)	(0.05)
Share of MIW in parliament	1.12**	0.26	-0.08	1.20**	0.27	-0.16
	(0.13)	(0.16)	(0.19)	(0.10)	(0.18)	(0.21)
Share of seats in parliament		0.94**	0.88**		1.02**	1.00**
		(0.16)	(0.16)		(0.18)	(0.18)
Constant	0.07**	0.08**	0.18**	0.08**	0.08**	0.21**
	(0.02)	(0.02)	(0.04)	(0.02)	(0.08)	(0.05)
R^2	0.72	0.81	0.71	0.64	0.76	0.60
No. of observations	680	680	197	680	680	197

Data source: Replication dataset for STA. A case is a party-in-government.
Figures in parenthesis are robust standard errors, clustered by country (as in STA).
*** = statistically significant at 0.01 level; * = statistically significant at 0.05 level*

Models B and C retrieve Warwick and Druckman's published findings, using the STA replication dataset. Controlling for raw legislative seat share dramatically reduces the effects of both theoretical voting weight and *formateur* status on portfolio payoffs (Warwick and Druckman (2006: 654). Model C confines the case universe to legislatures with five or fewer parties, in which MIWs and seat shares are not highly correlated. In this setting, the *formateur* effect loses statistical significance, although this is to a large extent the result of reducing the number of cases. This led Warwick and Druckman to infer that field data on portfolio allocation

do not allow us to infer a significant *formateur* effect once we control for the fact that *formateur* parties tend strongly to be large.[12] Models D-F re-estimate models A-C, subtracting the PM position from the dependent variable, thereby confining it to one side of the relevant regressions. While the other regression coefficients are robust to this change, the coefficient for *formateur* status is now effectively zero in all models. This allows us to infer that published empirical conclusions about the *formateur* effect in field data depend crucially on the endogenous coding of *formateur* status. STA's empirical findings are thus driven mechanically by the fact the "observed" *formateur* in their dataset is also invariably the eventual PM.

Setting out to address the problem of the endogenously coded "independent" variable at the heart of empirical tests of alternating offers bargaining models, we made sustained and determined efforts, using *Keesings*, to generate a new set of *formateur* codings that do *not* make use of the knowledge of the government that eventually formed, using only reports that relate to events prior to government formation. We concluded unambiguously that this is simply not possible, that a coding of exogenous *formateur* status, from any primary source that might be available, cannot be derived *without using information about the government that eventually formed*. It is theoretically conceivable that the recognition automaton is using a dog whistle to communicate its random picks to party leaders, and only party leaders, leaving nothing on the record. But it is not in practice systematically possible to observe *ex ante* exogenous *formateur* status in primary data sources. Methodologically, this implies BF-style models are not testable using a variable for exogenous *formateur* status, coded from historical sources. Theoretically, it raises the possibility that the assumption of an exogenous recognition automaton with common knowledge output is false, and that *formateur* selection is *endogenous* to the government formation process, a possibility we return to below.

3. THEORETICAL PROBLEMS OF EXTENSION TO *N*-PARTY WEIGHTED VOTING

Given the empirical claim that MIWs, rather than party seat shares, should be used to predict portfolio allocations, it is very striking that neither STA's definition of voting weights and resulting party types, nor any explicit feature of their formal proofs, constrains voting weights to be MIWs. Their definition of voting weights (STA: 984) constrains these only to be positive integers, true for both raw seat shares and MIWs. No argument deployed by STA uses any specific feature of MIWs and their proofs can equally be read as taking "weights" to mean raw seat shares. They introduce MIWs only after all core results have been proved: "*in what follows* we will use minimum integer weights" (STA: 988, emphasis added). This implies STA's propositions, if true, are simultaneously true in a given case for different types of voting weight, including both raw seat shares and MIWs. This in turn implies axiomatically these propositions are false. Equilibrium continuation values, indeed any quantity, cannot simultaneously be

[12] *Formateur* parties in the STA case universe have a mean seat share of 0.344, non-*formateur* parties of 0.118, a difference of means statistically significant at well beyond the 0.0001 level.

proportional to two different sets of weights that are not proportional to each other.[13] If we take party weights as MIWs, STA seem to have proved propositions they set out to test empirically, *which they contrast with Gamson's Law*. If we take party weights as raw seat shares, they appear to have proved a version of Gamson's Law. Both things cannot be true.

It is useful here to introduce some notation, following STA. In a legislature N, with a total number of votes w, each legislator i is of type $t \in T$, with all legislators of the same type having the same voting weight, w_t. Let $t(i)$ denote the type of i, n_t the number of legislators of type t present in N, and v_t the continuation value of a legislator of type t. For a *formateur* i of type t and a proposed coalition C, $\underline{v}_t$ is the minimum total price paid by the *formateur* to its partners in C; thus $\underline{v}_t = min\ v(C\backslash i)$.[14]

Difficulties generated by non-homogenous voting games

The "proving" of contradictory propositions about the same thing arises from the very distinctive "replication" technique in STA's proof strategy, adopted to deal with non-homogenous games, in which all MWCs do not have the same aggregate MIW. A recognized *formateur* in a non-homogenous game must choose between MWCs of different weights. Some non-dummy parties in non-homogenous games, furthermore, may be members of no *smallest-weight* MWC, generating significant ambiguities when analyzing BF-style bargaining models. Consider the non-homogenous weighted voting majority rule game with voting weights (4, 3, 3, 2, 2) and a quota of 8. If the leader of the largest party is recognized as *formateur*, s/he can choose as partners: the two small parties (forming an MWC of weight 8); one small and one medium party (MWC of weight 9); the two medium parties (MWC of weight 10). Does s/he see all these MWCs as equivalent, or see MWCs with different weights as being "different"? If s/he sees them as equivalent, then continuation values of the medium and small parties must be the same, since these parties are perfect substitutes for each other as partners for the largest party. A simple extension of this argument gives all parties equal continuation values, of 1/5. If s/he has read STA's papers and expects continuation values to be proportional to voting weights, then s/he will see coalitions with the two smallest parties as the "cheapest" alternative yielding the highest retained surplus, and strictly prefer the two small parties as coalition partners when recognized as *formateur*. If recognized *formateurs* do believe STA's propositions that continuation values are proportional to weights and strictly prefer the smallest MWCs however, we show in the Appendix that this in turn implies continuation values *decrease* monotonically in weights.[15] STA's core propositions cannot be equilibrium beliefs for recognized *formateurs* in this non-homogenous game. Either continuation values are all equal, regardless of party

[13] It is possible STA *implicitly* refer to MIWs in their proofs. But no feature of these proofs uses any specific property of MIWs, as opposed to any other type of voting weight such as raw seat shares, that agents might have in mind when bargaining over government formation

[14] We retain STA's notation, using w for voting weight, because STA do not distinguish between raw seat shares and minimum integer weights. This is obscured by the use of examples expressed in MIW format. We adopt STA's usage of w_i when the distinction between l_i and m_i has been left undetermined.

[15] Ordering parties by size, continuation values in this case are (2/16, 3/16, 3/16, 4/16, 4/16).

weights; or they are monotonically *decreasing* in party weights.[16] Neither of these outcomes matches STA's intuition based on economic theory; the largest party has an expectation either the same as, or *lower than*, one of the smallest parties.

For homogenous games, there is no problem assuming *formateurs* evaluate all MWCs (which by definition all have the same weight) as identical, regardless of individual weights of parties that comprise them. There is no ambiguity in equilibrium beliefs about continuation values of potential coalition partners, since only parties with the same weight are perfect substitutes for each other in MWCs. Thus, constraining results to strong[17] homogenous games, Montero independently proved continuation values in BF-style bargaining games are proportional to voting weights, assuming recognition probabilities proportional to voting weights (Montero 2006). As we have illustrated with the above example, however, there is considerable unresolved ambiguity in non-homogenous games about what recognized *formateurs* might believe about MWCs with different aggregate weights. On different (essentially behavioral) assumptions, parties with different weights may, or may not, be seen as perfect substitutes for each other in MWCs of which the recognized *formateur* is a member.

"Replicated" weighted voted games

Seeking results that extend beyond strong homogenous games, STA analyze *replicated* games, in which the number of players with each different MIW is multiplied by some positive integer, *r*, yielding a new game with many more players. Core propositions are proved by STA for "suitably chosen" *r* – from a range of integer values with no effective upper bound (STA: 999).

> We address this problem by examining the behavior of "replicated" voting games. That is, we examine equilibrium strategies as the number of players of each type is multiplied by some positive integer, $r \in \mathbf{Z}_x$. The basic game described above has $r = 1$, and a game with r replications has rn players, a total weight of rw, and a threshold for victory of $r\underline{w}$. We show that the effect of nonhomogeneity becomes small as r increases, thus allowing us to derive some general results. (STA: 984-985)

The replication device, introduced to deal with non-homogenous games, is used for all proofs, which make no distinction between homogenous and non-homogenous, or between strong and non-strong, games. Two serious problems arise with a proof strategy that replicates the game of interest *r* times. First, the replicated game typically has a bargaining structure completely

[16] The computer program distributed by STA in association with their paper produces the same results as our calculations in the Appendix.

[17] A strong game is one in which the complement of every losing coalition is winning. Complications posed by non-strong games for BF style bargaining models are: the need to model what happens in the event of blocking coalitions; the possibility there are pairs of parties that are never in the same MWC – impossible for strong games. Consider for example the non-strong majority voting game (3, 2, 2, 1) for which $Q = 5$. The largest and smallest parties share membership of no MWC. (We thank Maria Montero for this point and example.) Many published bargaining models implicitly assume strong games by assuming a simple majority quota and an odd number of legislators. Non-strong games are common in real legislatures. For the record, 132 of the 329 legislatures analyzed by STA in their published results generated non-strong games.

different from its $r = 1$ version. While voting weights may be replicated, *the set of winning coalitions is not*. Completely new types of coalition emerge in replicated games, while other types of coalition may disappear on replication. Thus even the most elementary of all strong homogenous voting games, (1, 1, 1), becomes the radically different *non-strong* game (1, 1, 1, 1, 1, 1) when $r = 2$. It is easy to see why the "effect of non-homogeneity becomes small as r increases". Consider the *strong non-homogenous* game (2, 2, 2, 1, 1, 1) with a quota of 5. The $r = 2$ version of this game is (2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1) with a quota of 10; this is a *non-strong homogenous* game. Setting $r = 2$ in this case does not so much "reduce the effects of non-homogeneity" as it defines a completely new *homogenous* game using the same agent types.[18] Self-evidently, for example, *any replicated strong game for which r is even is non-strong*. There is no result in the STA paper that demonstrates different legislatures formed by different values of r are in an equivalence class. Nor is such a result possible. Sets of winning coalitions and thus the likelihood of different coalitions differ across replicated legislatures, as do recognition probabilities. The expected surplus for the *formateur* changes as the game is replicated. In the simple (1, 1, 1) game, the *formateur* retains: 2/3 when $r = 1$; 1/2 when r = 2; 5/9 when $r = 3$. Valid logical inferences about the $r = 1$ game which is the subject of interest cannot be drawn from a hypothetical new game in which $r > 1$, introduced to facilitate analysis; this is a completely different game.

The second problem arising from replicated games shows how we can "prove" propositions simultaneously consistent with different things – continuation values simultaneously proportional to raw seat shares and MIWs, for example. This follows from Lemma 1 (STA 996-7), used in two subsequent lemmas (STA 998-9), the resulting three lemmas repeatedly used in the proofs of core propositions. Lemma 1 deals with a crucial quantity for BF-style bargaining models, the "cheapest" offer a recognized *formateur* can make to partners in winning coalitions. STA label this quantity for a type t agent as $\underline{v}_t$ and their proofs depend on setting an upper bound on this quantity. The lemma states that, in a stationary equilibrium and for any real $\varepsilon > 0$, there exists a finite r_ε such that for any $t \in T$ and $r \geq r_\varepsilon$, it follows that $\underline{v}_t \leq (rQ - w_t)/(rw) + \varepsilon$.[19] Setting ε arbitrarily small and rearranging, this gives us the result that it is possible to find a large enough r such that $\underline{v}_t \leq Q/w - w_t/rw$. Clearly, as $r \to \infty$, $\underline{v}_t$ becomes arbitrarily close to a constant, Q/w, for any w_t and approaches ½ for simple majority games. Equally clearly, "suitable" values of r can be chosen to set very different upper bounds on $\underline{v}_t$. This shows us why the core STA propositions can be simultaneously true for different sets of weights. The upper bound on $\underline{v}_t$ is determined solely by the expression w_t/rw in the limit. The same upper bounds can be derived for different values of w_t by choosing "suitably different" values of r.

[18] The list of examples is easily extended. Consider a classic homogenous majority rule "apex" game such as (2, 1, 1, 1). The $r = 2$ version of this games is (2, 2, 1, 1, 1, 1, 1, 1) and is clearly not an apex game. Any apex game, by definition, ceases to be an apex game on replication.

[19] STA use the notation $\underline{w}$ for the quota, in place of our Q.

4. TOWARDS A NEW MODEL OF NEGOTIATION OVER GOVERNMENT FORMATION.

We heartily applaud STA's ambition to extend the canonical model of legislative bargaining to arbitrary *n*-party settings with weighted voting, in a way that deals with non-homogenous and non-strong games.[20] Without this, we find ourselves in the odd situation that mainstream models of bargaining in legislatures cannot help us understand one of the most important substantive manifestations of this – bargaining in multiparty legislatures over government formation. Compounding the difficult *theoretical* issue of modeling weighted voting games in arbitrary *n*-party systems is the difficult *scientific* issue of specifying a bargaining model that can be "tested" using field data generated in a variety of local institutional settings. As Diermeier and Krehbiel persuasively argue, while non-cooperative game theory has the great advantage that it "explicitly models some features of political institutions and thus highlights how and why institutions matter … [o]ne may argue that the explanatory power of game theory is also limited by the very same features. Many games have multiple equilibria and sometimes the analysis seems to depend too much on the details of the game form, especially in bargaining models" (Diermeier and Krehbiel 2003: 138).

Thus the *formateur* bonus implied by many BF-style bargaining models is in effect an outcome bestowed entirely by an exogenous random recognition automaton, but this automaton is figment of the modeler's imagination, a technical assumption about the game form, not a substantive assumption about the real world. More generally, the very many different possible bargaining protocols that might be assumed in such models can generate quite different theoretical results, while real bargaining protocols are likely to differ from setting to setting. Furthermore, as Diermeier, Eraslan and Merlo (2003: 31) point out, "constitutions are typically silent with respect to the rules for selecting a formateur, which are generally reflected in unwritten conventions and norms. This is the case for all countries we consider". This calls into empirical question the fundamental grounding assumption of an *exogenous* recognition automaton with common knowledge output. And, as noted above, we ourselves have found that *ex ante* identification of a common knowledge monopoly *formateur* is not reliably possible using standard historical sources.

Taken together, all of this suggests the need for fundamental reconsideration of how to model bargaining over government formation in *n*-party legislatures. The enormous variety of actual or possible bargaining models that follow the Rubinstein-BF approach is something of a curse. In effect we are drowning in a sea of bargaining models, even confining ourselves to those specified and solved rigorously in their own terms. The fix, we argue below, is to specify bargaining models based on premises that can plausibly be argued, on substantive grounds, to have empirical relevance to the environment they characterize.

[20] Authors' calculations based on STA replication dataset show that only 176 of the 329 legislatures analyzed generated strong homogenous games.

Institutional constraints on government formation in parliamentary democracies

We thus agree with Diermeier and Krehbiel (2003), and with Diermeier, Eraslan, and Merlo (2003), that the best way forward is to model key institutional features of actual parliamentary systems and test these models against available data. In this section, therefore, we set out a "meta-model" of government formation in parliamentary democracies, premised on a set of strong, binding, and general constitutional constraints on this critical political process. Our approach is particularly appropriate in relation to bargaining *over government formation*, since a defining characteristic of democratic governance is that unambiguous constitutional rules specify how legitimate governments gain and retain office. Real constitutions always take *extravagant* care to preclude situations in which there is no legal government. Thus, if we are to improve our models of legislative bargaining in parliamentary systems, our best hope is to incorporate core constitutional features of these systems into our models.

Four features are of particular importance in this context. The first is that there is *always* an incumbent government. More precisely:

> C_1: *The constitution requires that the incumbent government remains in place until formally replaced by an alternative.*

This is neither a technical modeling assumption nor a strong behavioral regularity; it is a binding constitutional constraint. It applies even if the incumbent is a *gouvernement démissioné* (caretaker) that has been defeated in the legislature or has resigned. There is variation from country to country in how much a *gouvernement démissioné* can embark upon "new" policy initiatives (Laver and Shepsle 1994). But the constitutional bottom line is always that incumbent government ministers, and policies implemented by the incumbent government, remain *in situ* during government formation. Government formation always takes place in the context of a *status quo* government with different implications for different agents. This situation can last a long time. The Belgian government sworn into office on 20 March 2008, to take an extreme example, emerged from negotiations that started after the general election of 10 June 2007, during which the "outgoing" Belgian cabinet remained in office and life in Belgium continued as normal.[21]

The possibility of a *gouvernement démissioné* – a government that has resigned or been defeated but nonetheless remains the incumbent until replaced – focuses attention on how the government formation process was triggered. This is often ignored by theorists of government *formation* but is of prime concern to scholars analyzing government *termination* (Diermeier and Stevenson 1999; Diermeier and Stevenson 2000; Lupia and Strom 1995). For obvious reasons, government termination is also subject to explicit and binding constitutional provisions. Indeed, in the absence of such provisions, a polity is not a democracy. We identify three important

[21] It is possible to imagine possible BF-style bargaining models that allow for an incumbent government benefitting from any delay in negotiating its own replacement, although these would be considerably more complicated than most extant BF-style models.

constraints in relation to government termination: the requirement for regular elections; the possibility of an early dissolution of the legislature in most parliamentary systems; and the defining constitutional constraint in parliamentary systems, that the executive must retain the confidence of the legislature. Once again these are binding constitutional constraints, not "mere" modeling assumptions or behavioral regularities. More precisely:

> *C_2: There is a constitutionally specified maximum period between elections.*
> *C_3: The constitution provides for an incumbent PM to request early dissolution of the legislature and new elections; this request will ultimately be granted.*
> *C_4: The constitution requires that the incumbent government, and any putative alternative, must be able to win legislative confidence votes.*

C_2 is an integral part of any operational definition of democracy and applies in every country to which models of bargaining between parties over government formation are applied. It has long been recognized by students of government termination that government durations are "censored" by this constitutional regularity (Diermeier and Stevenson 1999; Diermeier and Stevenson 2000; King et al. 1990; Warwick 1994). This in turn implies directly that some government terminations, thus subsequent government *formations*, are exogenously triggered by a constitutionally mandated inter-electoral period.

C_3, providing for early dissolutions, is a pervasive constitutional feature of parliamentary government, though is not logically intrinsic to this. In practice, most elections in most parliamentary democracies are "called" by the incumbent Prime Minister (PM), subject to the constitutional maximum specified in C_2. This creates an environment with endogenous election timing, in which the incumbent PM has a constitutionally embedded distinguished position as the only person with the ability to choose between the current legislature and the legislature that would arise in the event of an early election (Smith 2004).[22]

C_4, the constitutional requirement that the executive retains the confidence of the legislature, is the unambiguous defining constitutional feature of parliamentary government. If C_4 does not hold there is not a parliamentary government system. As part of the formal process of forming a new government, we find legislative *investiture* votes in some parliamentary democracies. For countries with no formal investiture vote, the crucial constitutional fact of life concerns the ability of any incoming government to win an immediate majority vote of (no) confidence that has a constitutionally privileged pace on the legislative agenda.

Putting constraints $C_1 - C_4$ together, we generate the dynamic institutional meta-model of government formation set out in Figure 2. Any more precise model of government formation, grounded in more detailed institutional and behavioral assumptions, must be set within the unambiguous constitutional constraints described in Figure 2, which captures the fundamental point that any incumbent government is a dynamic equilibrium in a complex system. This

[22] It may be that a head of state can refuse such a request; the extent to which this happens varies considerably between countries. But such refusal effectively makes the head of state a veto player in government formation, a complication we do not get into here.

system cycles states around the loop C_1, C_2, L, C_1, unless one of two quite distinct things happens to trigger a government formation process. Either there is an election and a new legislature; or unmodeled random events perturb model parameters so that some legislative majority now prefers some alterative government.

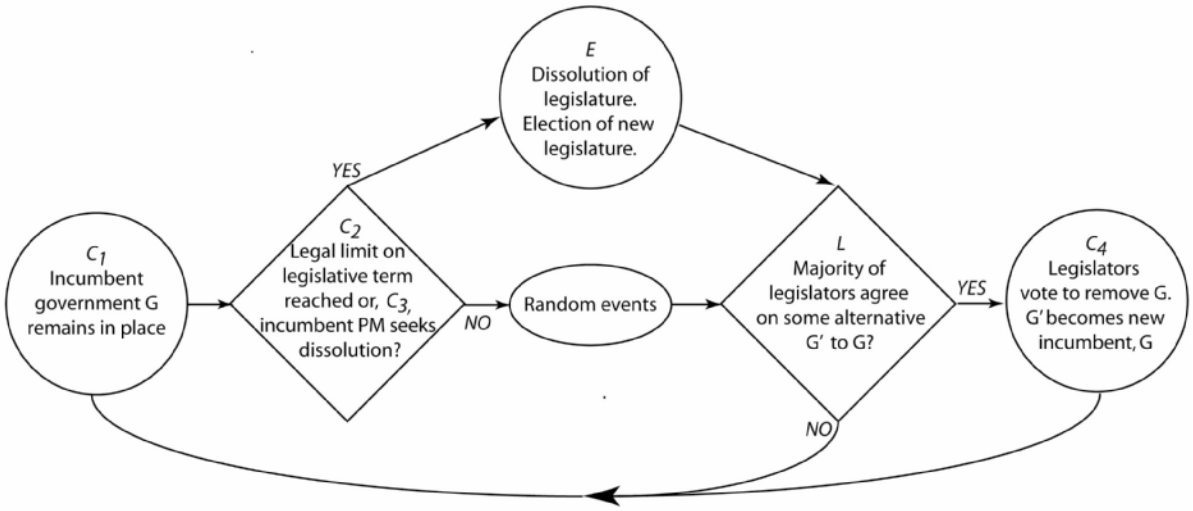


Figure 2: Institutional meta-model of government formation in parliamentary democracies

This characterization is compatible with an influential model of government *termination* specified and analyzed by Lupia and Strom (1995: 648), then developed and investigated empirically by Diermeier and Stevenson (1999, 2000). These authors do not model the complex system set out in Figure 2, but one-shot episodes of government negotiations, triggered by the arrival of a "critical event" (Lupia and Strom, 1995: 653). The types of event they have in mind change the electoral expectations of pivotal agents, thereby changing incentives to bring down the incumbent government and/or trigger elections. Taking additional account of policy preferences of pivotal actors, it is also possible to imagine events that shock the policy space and thereby destabilize government equilibria (Laver and Shepsle 1998).

Since every government formation must be preceded by a government termination and since, constitutionally, there are two distinct types of government termination, there are two distinct contexts for government formation. One is triggered exogenously by the constitutional requirement to hold regular elections; the other is triggered endogenously as a result of strategic decisions by pivotal agents. While most models of government formation assume, albeit implicitly, an exogenous trigger, this is not the typical case*; the majority of real cases of government formation are triggered endogenously*, with or without an early election. Information on types of government termination has been systematically compiled by Strøm, Müller, and Bergman (2008). Of all the government formations considered, about 40 percent were preceded by terminations coded by country experts as "technical", meaning that there was an exogenous constitutional or legal trigger. The remaining 60 percent of government formations were all triggered endogenously, either by "voluntary *early* elections" called by the

incumbent PM (about 18 percent) or by the incumbent government being terminated in what are coded as "conflictual" circumstances (about 42 percent).[23] In these latter cases, politicians bargained over government formation because at least one pivotal agent triggered the process endogenously, in light of some particular preferred outcome, and the real strategic action took place before the formation process was triggered.

The insight that government formation is heavily conditioned by the immediately preceding government termination is developed in recent empirical work by Martin and Stevenson that uses the Strøm, Müller, and Bergman data (Martin and Stevenson 2008; Strøm, Müller, and Bergman. 2008). Their firm empirical conclusion is that "termination context" does have a significant effect on subsequent government formation, in particular on the prospect of the incumbent government returning to office. Incumbency effects on government formation have been the subject of some empirical work (Glasgow, Golder, and Golder 2008; Warwick 1996) as well as some formal modeling in the BF tradition (Diermeier, Eraslan, and Merlo 2003). The core finding in Martin and Stevenson (2008), however, is that the probability of an incumbent returning to office is significantly reduced when the previous government has been brought down in "conflictual" circumstances, and significantly enhanced when the preceding government termination was either "technical" or the result of a voluntary early election. This makes perfect intuitive sense, of course, and is entirely consistent with the meta-model of government formation we set out in Figure 2.

Constitutional non-constraints on government formation

The binding constraints set out above are qualitatively different from the core premises of most BF-style bargaining models, which are not in any meaningful sense assumptions about the institutional structure of politics, but are essentially technical assumptions about the game form. Distinctive premises of these models include: (a) there is an exogenously selected common knowledge *formateur* with monopoly right to make proposals and; (b) "bargaining" consists of the recognized *formateur* broadcasting proposals which, if accepted by a legislative majority, are instantly and perfectly implemented by an exogenous automaton. It is possible to imagine a constitutional provision where *formateurs* are selected in order of the sizes of the parties they lead, though this is rejected on empirical grounds by Diermeier and Merlo (Austen-Smith and Banks 1988; Diermeier and Merlo 2004). Indeed, this provision is explicit in the constitution of Greece, though nowhere else we know about. It is, however, extraordinarily hard to imagine a constitutional provision that putative Prime Ministers are chosen at random, by an exogenous automaton using common knowledge probabilities. Both the *existence* of a privileged agent with a *monopoly* right to make proposals, and *random selection* of this agent, are highly unrealistic "brute force" assumptions used to justify employing a particular type of alternating-offers bargaining model. It seems much more reasonable to assume that government *formateurs* typically emerge endogenously, as part of the political game. This is an inconvenient assumption for alternating offers models, which depend heavily on exogenous *formateur*

[23] Authors' calculations using replication dataset for Strøm, Müller, and Bergman (2008)

selection, though Bassi has made progress with a formal model that involves endogenous selection of the *formateur* (Bassi 2008).

A closely related and equally unrealistic technical modeling assumption is that a single agent has the monopoly right to make publicly broadcast offers at any given point in the "bargaining" process, and that there are no private negotiations. Two quite different things are often confused in this regard. On one hand are formal rules of *legislative procedure*; on the other are formal rules, if any, constraining *bargaining between senior politicians over government formation.* As a matter of formal procedure in the chamber, legislators do not (usually) all speak at once but are recognized in sequence. At the same time it is far more plausible to think of the formation of governments as involving bargaining in the sense typically understood by civilians, meaning private *negotiation* between key players, as opposed to the sequential broadcasting of public offers. If we concede to orthodox bargaining models their own special use of the word "bargaining", we may usefully refer to *negotiation* over government formation as the unstructured interaction that takes place between principals in private.[24] *Privacy* is of the essence, since what is made public following negotiations is what negotiators *choose* to make public, nothing more. Following successful negotiations, principals walk out of a smoke-filled room and all claim victory. They also all claim agreement, a claim that is important in the context of another key constitutional constraint on cabinet government, the rule of *collective cabinet responsibility* for government decisions, which enjoins cabinet colleagues not to publicize their differences.[25]

Lack of structure is of the essence because negotiations over government formation involve the most experienced and sophisticated politicians playing for the highest possible stakes. Rather than assuming such people adhere to an unenforceable norm under which they make public offers in an exogenously choreographed sequence and engage in no back-room discussion, it is far more reasonable to assume *nothing* can prevent *any* politician from proposing *any* deal at *any* time during government formation. The deal emerging from the rough and tumble of such negotiations must be taken from the smoke-filled private room to the formal public world of the legislative chamber; sophisticated politicians of course anticipate this journey which itself is part of the game. But the government formation game is never, in any vaguely meaningful sense, actually played according to formal rules on the floor of the legislative chamber which is where, in reality, the outcome of the government formation process is simply announced.

It is of course easy to understand why formal theorists might fear abandoning the structure provided by the modeling assumption of sequential offers. If simultaneous offers are allowed, it is possible to show that many forms of bargaining games become isomorphic with Colonel Blotto games. Blotto games have very large sets of equilibria and most theorists have followed BF in preferring games with "sharp predictions" based on equilibrium refinements. It

[24] It is striking that civilians typically regard "bargaining" and "negotiation" as synonyms, while orthodox theorists of legislative bargaining tend to regard them as antonyms.

[25] Formally, cabinet ministers must resign (or be sacked by the PM) if they cannot publicly assert agreement with all cabinet decisions.

is our opinion, however, that a better way to generate sharp predictions is to develop institutionalist models premised on constraints that are imposed by real institutions, rather than on technical assumptions such as exogenous automata, and an imagined sequence of play.

5. WHAT IS TO BE DONE?

Moving beyond empirical and theoretical failures of orthodox alternating-offers models of "bargaining" in legislatures, at least as these apply to government formation in parliamentary democracies, we specify the dynamic constitutional meta-model of government formation set out in Figure 2. We find empirical support for this in recent work that highlights ways in which the context of government termination conditions the subsequent government formation process. Precise formal models specified within this set of general constitutional constraints will depend on specifying utility functions for key agents, and a more detailed formal characterization of the government formation process. Utility functions for politicians engaged in governmental politics are likely to contain components for private perquisites of office, public policy outputs, and some trade-off between the two. Characterizing the government formation process, an essentially empirical task, presents more formidable challenges.

Early models of government formation, reviewed by Laver and Schofield (1998), were essentially axiomatic, and in the traditions of cooperative game theory. Noting that important equilibrium results from cooperative game theory may be retrieved by analogous non-cooperative models[26] (Austen-Smith and Banks 2005; Moulin 1980), an axiomatic approach to modeling government formation in a comparative context also has strong substantive attractions. Foremost among these is that government formation is the biggest game in town. Since the stakes are so very high, and since the players are arguably the most sophisticated in the business, it does not seem plausible to assume that any but the most explicit and powerful constitutional constraints can be binding. This is why we set out constraints $C_1 - C_4$ and the constitutional meta-model these imply. And this is why, even if all issues with alternating-offers bargaining models were addressed, we would on substantive grounds reject this paradigm for modeling bargaining over government formation. It just does not seem in any way plausible that the "binding" institutional structure on which this paradigm is premised, the existence of an exogenously choreographed sequence of randomly selected monopoly proposers, could conceivably constrain real politicians forming real governments. Using an axiomatic approach thus captures the intuition that, for such a very important political process, many of the "rules" of the government formation process are essentially endogenous.

The alternative is to try building the binding constraints we have set out into a non-cooperative model of negotiation between party leaders over government formation. This will force us to resolve the implicit theoretical trade-off set up by Diermeier and Krehbiel (2003). Non-cooperative game theory tells us that local institutional details matter. Local institutional

[26] In the present context, and confining results to strong homogenous games, for example, it is possible to show that the non-cooperative model set out by STA can be solved as a system of axiomatic constraints (proof available from the authors).

details of the government formation process are different in every country. A model that gives us insight into the substantive phenomenon, vital in a comparative context, of negotiation between party leaders over government formation thus needs to be precise enough to capture local institutional variation, yet general enough not simply to be a model of government formation in Germany, or Spain, or Italy, or wherever. However this is achieved, the local institutional detail that structures such models must come from genuinely binding institutional constraints in the substantive environment modeled, not from "institutions" that are, when all is said and done, just tools in the modeler's box. Moving forward, therefore, the rigorous modeling of negotiation between party leaders over government formation in parliamentary democracies will force us to confront deep and difficult epistemological issues about the type of theoretical model we should be building.

APPENDIX: CONTINUATION VALUES NON-MONOTONIC IN VOTING WEIGHTS FOR A NON-HOMOGENOUS BF-STYLE BARGAINING GAME

Consider a majority rule legislature (4, 3, 3, 2, 2) with Q = 8. There are three different party types: w_1 = 4; w_2 = 3; w_3 = 2. Let μ be the set of MWCs and μ_{min} **be** the set of MWCs with smallest aggregate weight. There are four coalition types in μ: $\tau_1 = (t_1, t_3, t_3)$; $\tau_2 = (t_2, t_2, t_3)$; $\tau_3 = (t_1, t_2, t_3)$; $\tau_4 = (t_1, t_2, t_2)$. However, only τ_1 and τ_2 are in μ_{min}. The smallest party is a member of all coalition types in μ_{min}. STA's Propositions 2 and 3 are derived by solving the non-cooperative bargaining game for equilibrium continuation values. This game can be solved as a system of constraints.[27] The first (budget) constraint is that the continuation values, v_t, of all five parties sum to unity: $v_1 + 2v_2 + 2v_3 = 1$.

The second (coalition) constraint is that the probabilities of any given *formateur* proposing one of the set of possible coalitions sum to unity. Noting both that we consider other possibilities in the text and that there are many alternative assumptions, assume here that recognized *formateurs* have read STA's core propositions and expect continuation values to be proportional to voting weights. They then strictly prefer what they believe are the "cheapest" coalitions, those in μ_{min}; the surplus retained from other MWCs is less. In this event the only coalitions *formateurs* propose, if recognized, are τ_1 or τ_2. Within μ_{min}, the t_1 party belongs only to τ_1 and the t_2 parties belong only to τ_2. The t_3 parties are the only ones belonging to both τ_1 and τ_2, so coalition constraints are simple. If a t_3 party is *formateur*, then the probability, c_1, that it proposes τ_1 and the probability, c_2, that it proposes τ_2 must sum to unity: $c_1 + c_2 = 1$

The third (substitutability) constraint arises from the fact that, in equilibrium, if one potential coalition partner has a higher "price", v_i, than another, then the *formateur* strictly prefers the cheaper partner. In equilibrium, *formateurs* will be indifferent between sets of coalition partners

[27]This is the solution approach adopted in the widely circulated computer program designed by STA to calculate MIWs and BF continuation values – the only program available to calculate MIWs in difficult situations.

with the same price. Only t_3 parties can chose between possible sets of partners, in coalitions in μ_{min}, yielding them the same retained surplus[28]: $v_1 + v_3 = 2v_2$.

The final constraint is that continuation values are subgame perfect. We must take into account the likelihood an agent will be chosen either as *formateur* or as a partner in another *formateur's* coalition. Assuming equal recognition probabilities for all agents, there are three continuation values[29]:

$$v_1 = \frac{1}{5}(1-2v_3) + \frac{2v_1}{5}\cdot(c_1)$$

$$v_2 = \frac{1}{5}(1-v_2-v_3) + \frac{2v_2}{5}\cdot(c_2) + \frac{v_2}{5}$$

$$v_3 = \frac{1}{5}(1-2v_2) + \frac{2v_3}{5} + \frac{v_3}{5}\cdot(c_1)$$

Solving this set of constraints produces the result:

$$v_1 = \frac{1}{8},\ v_2 = \frac{3}{16},\ v_3 = \frac{1}{4},\ c_1 = \frac{1}{2},\ c_2 = \frac{1}{2}$$

Contradiction of Propositions 2 and 3 in this case does not depend upon the inference from the STA model that only coalitions in μ_{min} will be proposed. If the set of coalitions considered is extended to μ, then each party type has three possible coalition types to choose between if selected as *formateur*. Not surprisingly, predicted bargaining outcomes are now completely different. Solving the constraints in this setting shows that each agent now has a continuation value of 1/5, again contradicting STA's propositions, as each partner is preferred equally *regardless* of their voting weights. In both cases, "continuation values" are non-monotonic in voting weights. Indeed parties' solved "continuation values" are either monotonically *decreasing* in their MIWs or equal regardless of weights, not directly proportional to them. The STA calculator noted above generates the same non-monotonic BF "continuation values" as our first case above. What this shows, of course, is *not* that we actually predict continuation values to be monotonically decreasing in their MIWs, but that STA's core propositions – continuation values monotonic in weights – are *not* equilibrium beliefs for recognized *formateurs* in this non-homogenous game. This is not an isolated case; we have identified considerable set of cases in which direct computations contradict STA's core propositions, resulting in equilibrium continuation values that are non-monotonic in MIWs.[30]

[28] In words, a 2-vote *formateur* can replace two 3-vote parties with a 4-vote and a 2-vote party.

[29] In words, the 4-vote party has a 1/5 probability of being formateur, proposing the (4,2,2) coalition, and retaining the surplus of 10/14, and a 2/5 probability that a 2-vote party will be chosen and, with probability c_1, offer it 4/14. There is a 2/5 probability that a 3-vote party will be *formateur*, in which case the 4-vote party has zero probability of receiving an offer since 3- and 4-vote parties only share non-SWCs. Constraints on v_2 and v_3 have analogous interpretations.

[30] For example, in addition to (4,3,3,2,2), the majority rule games: (5,4,3,2,2), (3,3,2,2,2), (7,6,5,2,2,2), (9,6,5,4,2,2), (6,5,4,3,2,2), (9,5,5,3,2,2), (7,6,5,4,2,2,2), (8,7,6,5,4,2,2,2)

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